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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

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SEPTEMBER 1902

The Density and Coefficient of Cubical Expansion of Ice — J. E. VINCENT — 129

An Experiment Relating to the Application of Lagrange's Equations of Motion to Electric Currents — WILLIAM S. DAY — 154

Sparking Distances Between Plates for Small Distances — R. F. EARHART — 163

The Generalization of Gauss's Theorem — S. J. BARNETT — 172

On the Cavendish Experiment and the Law of Inverse Squares in Electrostatics — S. J. BARNETT — 175

An Acetylene Generator — E. A. RANDS — 178

Test of the Liquid Air Plant at Cornell University — FRANK ALLEN and WILLIAM AMBLER — 181

The Heat of Vaporization of Air — J. S. SHEARER — 188

New Books: *Ostwald; De Morgan; Mach.* — 191

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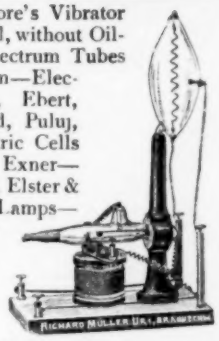

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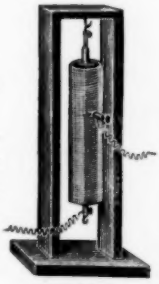

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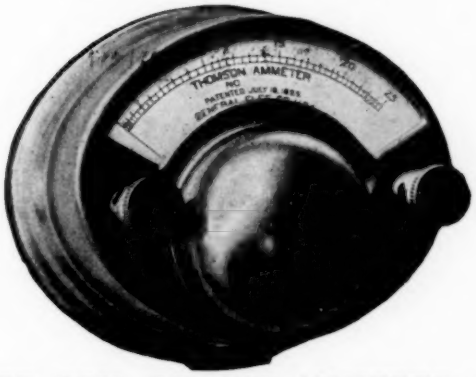

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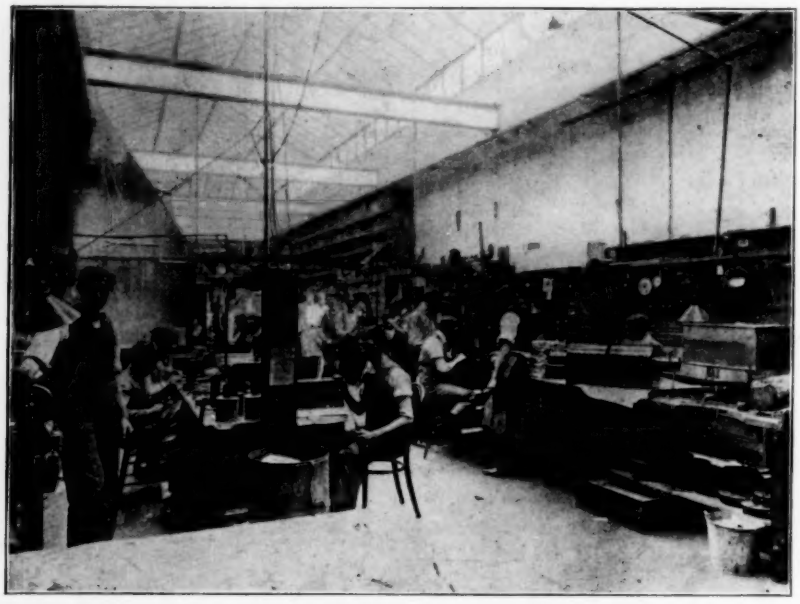

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THE

PHYSICAL REVIEW.

THE DENSITY AND COEFFICIENT OF CUBICAL EXPANSION OF ICE.[1]

By J. H. Vincent.

THERE are perhaps no subjects in the domain of experimental physics which call more urgently for attention, than investigations into the properties of water in its various states of aggregation. And of the various points which still need study, the latent heat of fusion is without doubt the most pressing. The method which promises to yield a reliable result for this determination requires a knowledge of the density of ice at 0° C.

The Bunsen ice calorimeter has, in the hands of Dieterici and other Continental physicists, recently become an instrument of precision, but the results which this apparatus is capable in itself of yielding, are unavailable to science owing to the lack of an accurate knowledge of the density and latent heat of ice.

But, as Griffiths[2] remarks, "there can be but little doubt that the mass of mercury expelled from a Bunsen calorimeter by the subtraction of a definite thermal unit, is a quantity that can be and doubtless will be determined with accuracy." It was with the object of contributing something to the solution of this problem that the investigation to be detailed subsequently was undertaken.

[1] Communicated to the Royal Society of London, February 6, 1902.

[2] Griffiths, Philos. Trans., A, 186, p. 265, 1895.

PREVIOUS METHODS AND RESULTS.

The first paper of importance, as regards scientific accuracy, on these subjects was published by Brunner in 1845;[1] but before treating of his paper, we may glance at the state of knowledge on the subject when he attacked it. He was led to take up the research by the fact that Petzholdt[2] had announced that ice expanded when its temperature was lowered. Petzholdt obtained this result experimentally, and proceeded to found thereon a new theory of glacier action which had the effect of bringing his paper into prominent notice. The idea that ice contracted on warming was an old one, and had been originally mooted by Musschenbroek, a hundred years previously.[3] Mairan[4] also supported it by experiments published ten years later. But Heinrich[5] in 1807 had obtained a positive coefficient. His result, obtained by the direct determination of the change in length of a bar of ice, yields the value .000024 as the linear coefficient for a degree centigrade. This observer also found the density of ice to be .905. Thus the subject stood when Brunner commenced his experiments.

Brunner started experimenting in the direction of preparing air-free ice from boiled distilled water, but failed to obtain it free from air bubbles. Even when he covered the surface of the water with turpentine immediately after boiling, the product had still to be rejected owing to its being full of small cracks; so that he was led to use selected pieces of river ice.

The method consisted in weighing the ice in air, and in either turpentine or petroleum, which latter liquid had the advantages: (1) of its smaller density; (2) its freedom from solvent action. He determined the density of the liquid by weighing a piece of glass in it immediately before and after weighing the ice, and he subsequently weighed the same piece of glass in water at different temperatures. He satisfied himself by direct experiment that by determining the temperature of the oil he also obtained the temper-

[1] Brunner, Pogg. Annalen, 140, p. 113, 1845.
[2] Petzholdt, Beiträge zur Geognosie von Tyrol, 1843.
[3] Musschenbroek, Essai de Physique, Leyden, 1739.
[4] Mairan, Dissertations sur la Glace, Paris, 1749.
[5] Heinrich, Gilbert's Annalen, 26, p. 228, 1807.

ature of the ice suspended in it. The whole of the operations were conducted in a laboratory, the temperature of which never rose above freezing point. After making due allowance for the buoyancy of air, the result for the specific gravity of ice at 0° C., referred to water at 0° C., was .9180, or .9179 as the density in grammes per cu. cm. The linear coefficient of expansion was .0000375, which Brunner remarks was greater than that previously found for any other solid.

The paper of Petzholdt also set Struve[1] to work about the same time. He obtained the value .0000531 for the linear coefficient per 0° C., using long bars of artificial ice in his experiments.

Marchand,[2] using a dilatometer of glass containing mercury and the ice to experimented on, obtained .0000350 for the linear coefficient, but did not state the kind of ice used.

The dilatometric method was also employed in 1852 by Plücker and Geissler,[3] who determined the density and dilatation of artificial ice. They used a dilatometer of a remarkably elegant design (see Fig. 1). The cylinder, *M*, of thin glass is open at the bottom, and has a capillary tube, *c*, inside it. This tube is sealed into the cylinder, *M*, and also into the outer cylinder, *N*, at one end. At the other end of the outer cylinder was another capillary tube, and in the preliminary part of the experiment, this tube reached as far as *o* only. The coefficient of expansion of the glass was first determined by preliminary experiments with mercury. To partially fill the inner cylinder with water, a bulb provided with a small opening at the top was sealed on at *c*. This bulb was filled with distilled water, and after this had been boiled for some time the upper orifice of the bulb was sealed. During this operation the opening at *o* had been closed, but, after cooling, both the upper sealed point in the bulb and *o* were simultaneously opened. Water flowed into *M*, expressing an equal volume of mercury. The tube

Fig. 1.

[1] Struve, Pogg. Annalen, 66, p. 298.
[2] Marchand, Journal für prakt. Chemie, 35, p. 254.
[3] Plücker and Geissler, Pogg. Annalen, 86, p. 265, 1852.

at *o* was again closed, and the water in the fine capillary at *c* was displaced by slightly warming the apparatus. This caused the mercury, in which the central capillary dipped, to rise, and the bulb was then removed by sealing off at *c*. Finally the capillary tube *fdo* was sealed on at *o*, and the position of the end of the column of mercury marked on the tube after the whole apparatus had been reduced to 0° C. On freezing, the water in the inner glass vessel expanded and, breaking the inner cylinder, relieved itself from constraint.

The mean result for the increment in volume of unit of volume of water at 0° C., on changing to ice at 0°, was .09195, which is equivalent to .91567 for the density at 0°, while .0001585 was obtained for the coefficient of cubical expansion.

The next observations with which it is necessary to deal are those of Dufour.[1] Having previously experimented by finding the density of a mixture of alcohol and water in which ice floated in neutral equilibrium, he published in 1862 an account of experiments in which a mixture of chloroform and petroleum was used in preference to the former liquid, which dissolves ice. By taking the mean of 16 experiments, he obtained .9178, with a probable error of .0005, as the specific gravity referred to water at 0° C. He employed the value .000158 for the coefficient of expansion for reducing his results. The ice used was prepared from water boiled *invacuo*, and although free from air bubbles was "opaline" in appearance.

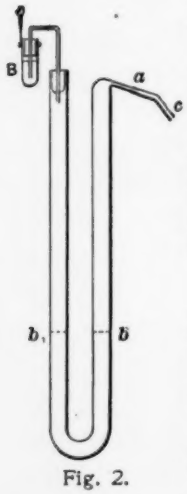


Fig. 2.

Bunsen's celebrated paper on Calorimetry[2] appeared in 1870. Amongst other researches included in this memoir is a determination of the density of ice at 0° C., by a dilatometric method which, according to the illustrious author, completely eliminated the errors which had rendered previous estimations uncertain.

Bunsen's dilatometer is shown in Fig. 2. It consisted of a thick-walled **U**-tube of hard glass drawn out at *a*, and this was filled

[1] Dufour, Comptes Rendus, 54, p. 1080.
[2] Bunsen, Pogg. Annalen, 141, p. 1, 1870.

with mercury up to the level, $b_1 b$, which was boiled for some time. Boiled water was sucked into the apparatus, and rested on the mercury at b. This water was then boiled in the tube for half an hour, the end a being, by means of a rubber tube c, led under the surface of water, which was also kept boiling. The dilatometer was allowed to cool, while the vessel into which c dipped was kept boiling; the side of the tube ab then became completely filled with air-free water, and the point a was then sealed off. By weighing before and after filling with water, the mass of water taken was obtained. The other limb of the U-tube was now filled with mercury, and the water was frozen by subjecting it to cold in such a way that the water froze from above downwards. The ice thus formed was absolutely clear. The cork and capillary tube shown in the figure were then inserted, and the whole apparatus surrounded with dry snow. The mass of the vessel B and its contained mercury was noted. On melting and again reducing to 0°, reweighing the vessel B provides the other datum requisite to compute the density of ice at 0° C. Bunsen's mean value was .91674.

No experiments on these subjects seem to have been published again until quite recent times, when Nichols[1] brought out his paper on the density of ice in 1899. Leaving the theoretical portion of this memoir out of consideration for the present, we find that Nichols determined the density of artificial and natural ice by several methods.

Fig. 3.

Method 1. Specific Gravity Bottle.—The apparatus consisted of a specific gravity bottle fitted with a tube (see Fig. 3), round which a cylinder of ice was formed. The unfrozen water was shaken out, and the whole again subjected to cold; by weighing in a laboratory, whose temperature was below freezing point, the mass of ice taken was found. The bottle was now filled up with cold mercury, the stopper inserted, and the whole left in an ice-bath over night with the stopper dipping in mercury. Finally, the stopper was dipped into a weighed quantity of mercury, the ice permitted to melt, and the whole apparatus again reduced to 0° C. The loss

[1] Nichols, PHYSICAL REVIEW, 8, p. 21, 1899.

of weight of the mercury into which the stopper dipped, gave the means of computing the density of the ice mantle at 0° C. free from any error due to deformation of the flask on filling with mercury. The result for the density from a single experiment was .91619.

Method 2. Brunner's Method.—Nichols employed refined petroleum, and weighed several varieties in it, again working in a laboratory below freezing point. The results were reduced to 0° C., by employing the value of .00015 for the coefficient of cubical expansion. The results obtained were :—

Kind of Ice.	Density at 0° C.	Kind of Ice.	Density at 0° C.
Artificial.	.91603	Natural (new pond ice).	.91792
Natural (icicles).	.91795	Natural (pond ice, 1 year old).	.91632

Method 3. Determination of the Volume of the Ice by Displacement. Nichols next attacked the question by the employment of an absolutely original method. An iron box of special construction, having a capacity of about 2 liters, and rectangular in shape, was nearly filled by means of a regular block of new pond ice, and the rest of its interior was filled up with mercury. From the weighings of this mercury and the ice, and a knowledge of the volume of the box, the density of the ice at 0° C. was computed. The ice contained a small quantity of air, the amount of which was separately determined and allowed for. The final value for the density came out at .91760.

Nichols[1] next turned his attention to the determination of the linear coefficient of expansion of ice. No work had been done on the dilatation of ice since 1852. The method employed was similar to that used by Struve in 1845. A bar of commercial artificial ice, which had been manufactured some months previously, was used, and Nichols again had the felicity of working in a laboratory which was never warmer than −3° C. during the work. The readings were obtained by measuring, by means of a dividing engine, the distance between the centers of two tiny drops of mercury resting in depressions in the ice about 40 cm. apart. The range of temperature was from −8° to −12° C. Four sets of readings were taken, with the mean result .0000540 for the linear coefficient.

[1] Nichols, PHYSICAL REVIEW, 8, p. 184, 1899.

In order to explain the remarkable discrepancies between the values obtained by previous observers, Nichols put forward the theory that in reality there are two kinds of ice which have been under experiment; the density of artificial ice being about .916, and that of natural ice more than one part in a thousand greater. This immediately throws the subject into a more tangible form, but the serious consequences of such a dual character for ice demand most careful consideration. It seems to me that if there are really two kinds of ice, differing in density so largely, these varieties would also have different latent heats, and, what is perhaps more important still, different melting points.

Nichols's theory is, however, supported entirely by his own work, and also by most of the results of previous observers. In this connection it must be pointed out that, according to one of Nichols's experiments, natural ice assumes a density approaching to that of artificial ice if the natural ice has been kept some time.

The value obtained for the density by Dufour for artificial ice is larger than that of other observers using the same variety. But the method of neutral equilibrium is far inferior in exactitude to the methods employed by Plücker and Geissler, Bunsen and Nichols. Neither must the results of Bunsen be accepted as being of extraordinary reliability in spite of his assurance that he had eliminated all error. The **U**-tube dilatometer suffers from the disability that any small difference in the method of holding it may cause considerable change in its voluminal contents. It must also be remembered that the ice in Bunsen's experiment was probably under considerable pressure, since each new layer as it was formed became the vehicle of transference of heat upwards from the underlying water. Ice is one of the most contractible of solids by fall of temperature, and thus when the whole of the water was frozen, it must have been considerably denser than it would be at 0° C. The sides of the somewhat narrow tube would tend to prevent the ice assuming its proper density as the temperature rose to 0° C. It should be noted that if the mean temperature of Bunsen's ice column was 4° or 5° below zero, this would suffice for the somewhat high value which he obtained. There is another and more serious objection to Bunsen's method. Any attempt to get ice exactly at 0° C. by sur-

rounding it with an ice jacket may result either in the resulting temperature being lower than 0° C., through the observer not allowing a sufficiently long time for the equalization of temperature or, on the other hand, may result in some of the ice melting. In either case the value obtained for the density will be too high. None of these objections apply to Plücker and Geissler's work.

Barnes[1] has recently determined the density of natural ice by weighing selected specimens in water. His results give the same density for old and new ice. The mean value obtained (expressed in grammes per cu. cm.) was .91649.

SYNOPSIS OF PREVIOUS WORK.

In order to facilitate reference, the results of previous workers have been set out in Table I., which gives the methods adopted, the variety of ice used, and the results obtained by different observers.

In Table II. the results for the two kinds of ice are separately set forth. The work of Marchand is omitted altogether from this table, as he did not state what kind of ice he used. The value of the density obtained for old pond ice by Nichols is also not included. The mean result for the density of *natural* ice at freezing point is .9176 gramme per cu. cm., while that of *artificial* ice is .9165 gramme per cu. cm.

If, however, we neglect Dufour's value, we obtain the result .9162 gramme per cu. cm. for artificial ice.

Only one estimation of the dilatation of natural ice is available. It is .0001125 for the cubical coefficient of dilatation for 1° C., while three results are available for artificial ice. The mean value is .000160 for the cubical coefficient of dilatation for 1° C.

Only one direct determination of the cubical expansion of artificial ice is to hand. This was obtained by Plücker and Geissler, and is .0001585 for the cubical coefficient of dilatation for 1° C.

In both Tables I. and II. the cubical coefficient only has been tabulated. In those cases in which the linear coefficient was actually determined, the cubical coefficient has been tabulated as three times the linear, but the legitimacy of this procedure is open to grave doubt in the case of a body like ice which is endowed with hexagonal symmetry of structure.

[1] Barnes, PHYSICAL REVIEW, 13, p. 55, 1900.

TABLE I.

Date.	Observer.	Method.	Kind of Ice.	Density. (grammes per cu. cm.)	Coefficient of Cubical Expansion
1845	Brunner.	Weighing in liquid.	Natural.	.9179	.0001125
1845	Struve.	Direct measurement of linear coefficient.	Artificial.	——	.0001593
1845	Marchand.	Dilatometric.	?	——	.0001050
1852	Plücker and Geissler.	Dilatometric.	Artificial.	.91567	.0001585
1862	Dufour.	Neutral equilibrium in liquid.	Artificial.	.9177	——
1870	Bunsen.	Dilatometric.	Artificial.	.91674	——
1899	Nichols.	Dilatometric.	Artificial.	.91619	——
		Weighing in liquid.	Artificial.	.91603	——
			Natural (icicles).	.91795	——
			Natural (new pond ice).	.91792	——
			Natural (old pond ice).	.91632	——
		Volume by displacement.	Natural (new pond ice).	.91760	——
1899	Nichols.	Direct measurement of linear coefficient.	Artificial.	——	.0001620
1901	Barnes.	Weighing in water.	Natural.	.91649	——

TABLE II.

Observer.	Natural.		Artificial.	
	Density.	Co. of Cub. Exp.	Density.	Co. of Cub. Exp.
Brunner.	.9179	.0001125	——	——
Struve.	——	——	——	.0001593
Plücker and Geissler.	——	——	.91567	.0001585
Dufour.	——	——	.9177	——
Bunsen.	——	——	.91674	——
Nichols.	——	——	.91619	——
	——	——	.91603	——
	.91795	——	——	——
	.91792	——	——	——
	.91760	——	——	——
	——	——	——	.0001620
Barnes.	.91649	——	——	——
Mean.	.9176	.0001125	.9165 or .9162 neglecting. Dufour.	.000160

PRINCIPLE OF THE METHOD EMPLOYED.

Since the question of the density of ice was still, in spite of all the labor that had been spent upon it, in a far from satisfactory state, and since a direct determination of the cubical coefficient of expansion had not been attempted since 1852, I was desirous of employing a method which should yield both results. In order that the work should have any value, it was necessary to employ some device other than any which had been used previously.

The method of weighing ice in mercury was one which naturally suggested itself. For the purpose of a sinker two metals are available, tungsten and platinum. Tungsten is difficult to work, but is readily procurable in any amount; if mercury attacks tungsten this could be avoided by protecting it by an iron shell. Although this direct method was not employed, I believe that the use of a platinum or tungsten sinker for weighing ice in mercury would be well worth the attention of future workers.

The necessity of a sinker can be done away with if the buoyancy of the ice is obtained by determining the tension of a wire which moors it to the bottom of the vessel. The tension of the wire may be found by passing it through a small hole in the bottom of the mercury-containing vessel, which latter must be closed at the top so that the mercury will not pour out of the hole through which the wire comes. This method obviates the use of a balance, for a scale pan may be hung on the wire, and equilibrium obtained by suitably adjusting the suspended weights. Joly has used this principle in the construction of a balance[1] and my apparatus differs from his, in that I introduce the material whose density is to be determined and use it as the float.

The advantages to be derived from the use of mercury are several. It can easily be obtained quite pure, is without solvent action upon either water or ice, and its coefficient of expansion is of the same order of magnitude as that of ice. Further, this coefficient is known with greater accuracy. The use of a liquid whose coefficient of expansion is near that of ice is helpful in determining the density at 0° C., and also in the determination of the coefficient of cubical expansion.

[1] Joly, Phil. Mag., September, 1888.

In order to determine the density at 0° C. it would be sufficient under ideal conditions of temperature to find the buoyancy of an inverted vessel in mercury, to introduce air-free water into this vessel, to determine the buoyancy of the vessel and also that of the water in its liquid and solid state. The experiments were performed by equilibrating the ice at temperatures below zero, and to find the density at these temperatures we must allow for the contraction of the mercury and of the vessel containing the ice. The equilibration of the water was always performed at zero.

Description of Apparatus.

The vessel which constituted the reservoir for the liquid used in the hydrostatic balance is shown in section in Fig. 4. It consisted of a funnel-shaped vessel, *A*, of cast iron. This was turned up in a lathe, and care was taken to have the inside surface quite free from "blow-holes." Into this vessel at *B*, a tube of steel, *N*, was driven. The lower end of this tube carried a tightly-fitting screwed piece *C*, the bottom of which was closed, except for a central hole about .4 mm. in diameter. This hole was for the wire to pass through downwards to support the scale pan, and upwards to tether the vessel which held the ice or water. The vessel shown in Fig. 4 may be called (to save circumlocution) the funnel.

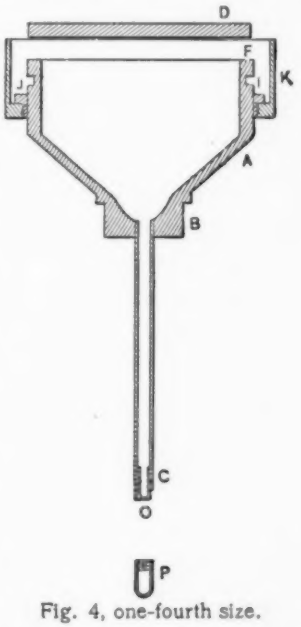
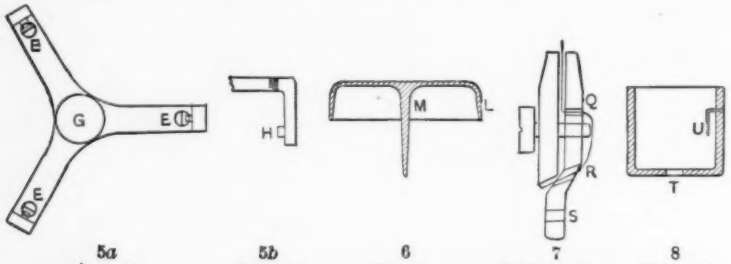


Fig. 4, one-fourth size.

The top of the funnel could be closed by an accurately fittingsteel circular plate, shown in section as *D*. In order to fasten this plate down securely on to the top of the funnel, a three-armed piece of iron, shown in Fig. 5*a*, was used. This was provided with three screws, *E*, the lower ends of which bore directly on the circular plate *D*, immediately over the annular plane-bearing surface, *F*. A central screw, *G*, with a milled head, served to hold the plate *D* in position in some manipulations which did not require that the sur-

faces at *F* should fit very closely. The three-armed piece was provided with three stout pegs, one of which is shown as *H* in Fig. 5*b*, which is a section of one of the portions of the tri-radiate clamp of which Fig. 5*a* is a plan. These pegs could traverse round the channel, *I*, Fig. 4, and their pressure on the upper roof of this chan-



5*a* (½ size) 5*b* (½ size) 6 (½ size) 7 (5 times size) 8 (½ size)

Figs. 5*a*—8.

nel, provided reactions for the pressure of the four screws *E*, *E*, *E*, *G*. In order to place the tri-radiate clamp in position, three small gaps were cut at angular intervals of 120° in the outside of the top rim of the funnel, and three similar gaps were provided in the plate *D*.

The funnel was provided with a screwed collar, *J*, which was permanently shrunk on to the cylindrical portion of its outer surface. This collar served to support a removable ring of iron, *K*. The walls of this ring (which we may call the mercury collar) were higher than the top surface of *D*, when this latter was in position. The mercury collar served two purposes; it provided a means of sealing the whole of the top of the funnel by flooding with mercury when the closing plate *D* was in position, the mercury it contained surrounded the bulb of the thermometer which was used to find the temperature of the contents of the funnel. When the funnel was not in position for the actual determinations of buoyancy, it could be held in a vise by the flat surfaces cut in the thick metal at *B*.

The vessel which served to contain the water or ice, while it and its contents floated in the mercury in the funnel, is shown in Fig. 6. It was somewhat of an umbrella shape, and was cut out of a solid block of mild steel in the lathe. It was perfectly smooth, and provided no lurking places for air. The sides, *L*, were made of decreasing thickness downwards as also was the central stem, *M*,

which was pierced at its lower end with a hole which served to attach the wire by which the scale pan was supported.

Steel wires of two diameters were used in the experiments—one, about .17 mm. in diameter, was used in the preliminary investigation of the dilatation of the umbrella, the other, about .2 mm. in diameter, was used in the actual experiments when the umbrella held ice or water. The wire passed through the mercury in the funnel, down the center of the tube, *N*, through the small hole in *C*, and had a specially constructed clamp attached to the end outside *C*. This clamp was made so that when held up close to the hole *o* by the buoyancy of the umbrella and its contents, it could be enveloped by the screwed closed tube, *P* (Fig. 4). The clamp is shown in Fig. 7. The peculiarity in the construction of this clamp was that it was pierced by the holes *Q* and *R*, through which the wire passed as well as being held by the jaws. This arrangement made the chance of the wire slipping very small. The course of the wire is indicated by a broken line in Fig. 7. The lower portion of the clamp was also drilled with a screwed hole, *S*. This hole served to receive a hook by which the scale pan used to hold the weights in the equilibrations was supported, and was screwed inside so as to enable the clamp to be attached to the piece *C* (Fig. 4) always at a definite distance by means of a second screwed piece which could be attached to *C*. The object of this arrangement was to ensure always that the wire was the same length.

In Fig. 8 an iron cylindrical vessel is shown in section. This reservoir could be put on the tube *N* (Fig. 4), which was made slightly conical at its lower end in order to fit into the hole *T*. A bent wire, *U*, was fixed into the side of the reservoir, and served to keep the stopper, *P*, submerged when the reservoir was full of mercury. The use of this portion of the apparatus will be referred to in describing the process of filling the funnel with pure, dry, air-free mercury.

The method of holding the funnel during the determinations, and the arrangements for surrounding it with ice or freezing mixture, are shown in Fig. 9.

The tube, *CN*, of the funnel, *A*, passed through an india-rubber bung, *V*, which closed the lower orifice of a large glass jar, *W*. The

funnel was supported by a framework of iron, *X*, which consisted of two rings joined by three bent wires. The lower ring rested in the concavity of the glass jar and bore the weight of the funnel. The jar in turn was supported by a larger rubber bung, *Y*, which fitted into a hole in the bottom of the lower wooden box, *Z*. Three stout brass pieces, *a*, only one of which is shown in the drawing, prevented the jar from tipping sideways.

The upper wooden box, *b*, rested on the lower box, *Z*, while the whole was surrounded on five sides by the non-conducting cases, *c*. These cases were removable boxes of wood loosely filled with cotton wool.

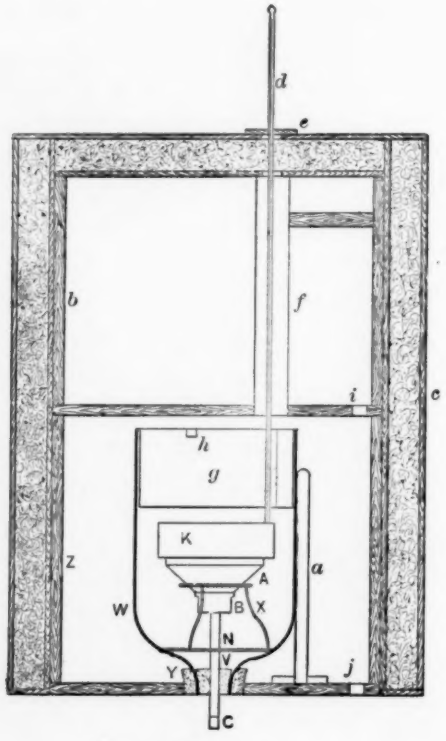


Fig. 9.

The thermometer, *d*, passed through a piece of ebonite, *e*, which rested on the top of the non-conducting case, through a hole in the case, down a wide brass tube, *f*, in the upper wooden box, and rested with its bulb in the mercury in the collar, *K*. The remov-

able thin metal vessel, *g*, was supported by three lugs, *h*, which were bent over so as to engage the rim of the glass jar. The upper and lower boxes were fitted with the holes *i*, *j*, which could be closed by bungs.

The whole of the apparatus shown in Fig. 9 rested on three levelling screws which in turn were supported by a strong table having a hole in its center through which the wire from the funnel passed.

The lower surface of the lower box was protected from the access of heat by filling in the space between it and the table with loosely packed cloth. The description of any other parts of the apparatus which may be necessary will be more conveniently given when dealing with the conduct of the experiments.

Determination of the Buoyancy of the Umbrella.—In order to find the buoyancy of the ice and water, it was necessary to determine that of the umbrella at different temperatures. If we know the load on the scale pan necessary to obtain equilibrium when the umbrella only is tending to float in the mercury, then the weights added when the umbrella and its contents are equilibrated, gives us the buoyancy of the contents. The funnel was taken and a thin steel wire (17 mm. in diameter) passed upwards through the hole *o* (Fig. 4) until it could be threaded through the hole in the stem of the umbrella, when it was fixed by twisting the end round itself. This operation was done is such a manner as to ensure that the length of wire thus used in the fastening was the same in each experiment. The wire was then pulled tight and the little clamp attached with its jaws about 1 cm. distant from *o* by the method previously mentioned. The whole length of wire used was thus always the same. The stopper, *P* (Fig. 4), was then screwed on *C* and mercury poured into the funnel.

The mercury used throughout these experiments was first cleaned by the ordinary chemical methods, then boiled in air, and distilled twice in a vacuum. After being used in one experiment it was filtered, boiled and distilled twice again before being used in a fresh determination.

The plate *D* was placed on the funnel and the whole was turned on one side to enable any air imprisoned under the umbrella to

escape. This was repeated until no more mercury could be poured into the funnel, when the latter was heaped up with mercury and the plate *D* was slid on to the top of the funnel and firmly screwed into position by means of the tri-radiate clamp. It was found necessary to carefully grind the bearing surfaces of the closing plate and of the funnel together before each experiment in order to obtain a perfect fit.

After thus filling with mercury, the funnel was removed from the vise and placed in an inverted position on a tripod. The stopper was then unscrewed from the tube, when the little clamp could be seen supported on the top of a straight piece of wire projecting a few millimeters through the hole at the end of the tube. The mercury reservoir (Fig. 8) was slipped on to the conical tube and the mercury in the funnel was then boiled by applying the flame of a large Bunsen burner to the closing plate. The mercury which came out of the small hole in this process, partially filled the reservoir, which on removing the flame was filled with mercury boiled in another vessel. The whole was left to assume the ordinary temperature, and then the stopper was inverted and plunged beneath the mercury in the reservoir, where it was prevented from rising to the surface by the wire *U*.

The boiling was again performed, and then the mercury in the tube and reservoir was also boiled. Operations of alternate heating and cooling were continued until I felt satisfied that no air or water remained in the funnel. The stopper was then taken from the wire in the reservoir, and manipulating it so as never to permit the fingers to come beneath its orifice, it was screwed on to the piece *C*, and the funnel was thus closed.

The reservoir was then removed, and the funnel was again put in the vise; this time with the mercury collar round it ready to be screwed on. The triradiate clamp and the closing plate were removed and the bearing surfaces were covered with a thin film of vaseline before pouring an excess of recently boiled mercury into the funnel and sliding the closing plate again into position. The plate was then fastened down securely and the mercury collar screwed on.

The funnel was then installed in the apparatus shown in Fig. 9, when the collar was filled with mercury and the top of the funnel

thus completely sealed. The stopper was then unscrewed and a small vessel of hot, recently boiled mercury was placed so that the hole *o* dipped beneath its surface. This caused the mercury in the tube to expand and to displace any air from about the orifice of the tube as the exuded mercury flowed through it.

The funnel was now levelled by adjusting the screws on which the bottom box rested, the level being placed with its legs on the top of the closing plate. The wire mooring the umbrella down then coincided with the axis of the tube. In order to find the buoyancy of the umbrella at 0° C., the vessel *g* was not used, but the jar and the boxes were filled with table ice, and the cases *c* placed round the apparatus. Cloths were packed under the lower box, and the whole was left over night.

When the equilibration was to be performed, the small mercury vessel into which *C* dipped was removed, the scale pan attached, and the weights adjusted so that equilibrium was attained when the wire projected 4 mm. from the hole. The reading having been taken, the pan was removed and the hole was again closed with hot mercury.

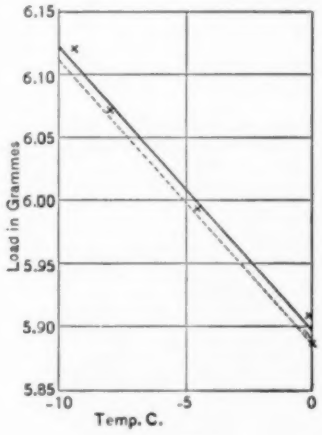


Fig. 10.

In order to get readings below 0° C., the ice was all removed, *g* was put in position filled with ice and salt, and the boxes were filled with the same mixture. Then the thermometer was inserted so as to have its bulb in the mercury collar.

The space round the funnel was clear of the freezing mixture, and the funnel thus changed its temperature so slowly that the thermometer readings could be relied on as giving the temperature of the funnel and its contents. All readings were taken with the thermometer slowly rising.

The thermometer used in these experiments was made by Hicks, and had the portion which projected above the cotton wool case graduated from 1° C. to 10° C. in tenths of a degree. I tested its accuracy at 0° C., and could find no error. The temperature rose

(after the apparatus had been left a day or so) about a degree in three hours, and readings of the buoyancy could be obtained at intervals.

The results for the weights necessary to be added to the pan, which itself weighed about 60 grammes, are set out in Table III., and shown also in Fig. 10.

It will be seen that the last weighing taken after five others agrees closely with the first, showing that no air gained access to the umbrella in the process of equilibration.

A new set of platinized weights by Oertling was used in these experiments. They were tested after the experiments, and were found to be consistent with themselves. Their absolute mass is not involved in the determination.

TABLE III.

Buoyancy of Umbrella.

Order of Weighing.	Circumstances.	Temperature.	Weights in Grammes.
1st.	After 15 hours in ice.	0°	5.890
2d.	After 21 hours in 1st freezing mixture.	−8	6.071
3d.	After 33 hours in 1st freezing mixture.	−4.48	5.995
4th.	After 45 hours in 1st freezing mixture.	−0.1	5.895
5th.	After 45 hours in 2d freezing mixture.	−9.42	6.120
6th.	After 72 hours in 2d freezing mixture.	−0.1	5.910

DETERMINATION OF THE DENSITY OF ICE AT DIFFERENT TEMPERATURES.

The wire and clamp used in equilibrating the water and ice were not the same as those used with the empty umbrella. The values for the buoyancy of the umbrella as read from the unbroken straight line on Fig. 10 are thus subject to a correction of .012 gramme, which must be subtracted from the values thus found. The results for the buoyancy of the umbrella are thus taken from the broken line in this figure.

Let W = the number of grammes necessary to equilibrate the water at 0° C.; *i. e.*, the load in the pan less the corrected buoyancy of the umbrella,

I = the number of grammes to equilibrate the ice at $-t°$ C.,

i_{-t} = the density of ice at $-t°$ C.,

w_0 = the density of water at 0° C.,

h_0 = the density of mercury at 0° C.,

h_{-t} = the density of mercury at $-t°$ C.

Then equating the two values obtained from the above for M, the mass in grammes of the material taken, we have

$$\frac{M}{w_0} h_0 - M = W \quad \text{and} \quad \frac{M}{i_{-t}} . h_{-t} - M = I;$$

therefore

$$\frac{w_0}{h_0 - w_0} W = \frac{i_{-t}}{h_{-t} - i_{-t}} I.$$

Let $$K = \frac{I}{W} \quad \text{and} \quad \frac{w_0}{h_0 - w_0} = q;$$

then

$$i_{-t} = \frac{q h_{-t}}{K + q},$$

which was the formula used in computing the results. Since the number K only depends on the ratio of the weights, no correction for the effect of the buoyancy of the air is necessary.

The density of water at 0° C. was taken as .999884, and that of mercury at the same temperature as .135956, while the density of mercury at lower temperatures was found from the formula of Chappuis[1] (Procès-verbaux des Séances du Comité International, 1891, p. 37). The results needed were read off from a plotted curve.

The funnel having been filled with pure dry air-free mercury in the manner already described, the closing plate was removed and air-free water introduced under the umbrella. This was accomplished by means of the apparatus shown in Fig. 11.

[1] Chappuis, Procès-verbaux des Séances du Comité International, p. 37, 1891.

This consisted of a glass bulb, *k*, which had a capillary tube, *l*, sealed into it above, and which terminated in a stout tube, *m*, below. The whole could be supported by fixing this tube in a clamp. A flexible rubber tube, *n*, communicated with the bulb and with a somewhat larger reservoir not shown in the figure.

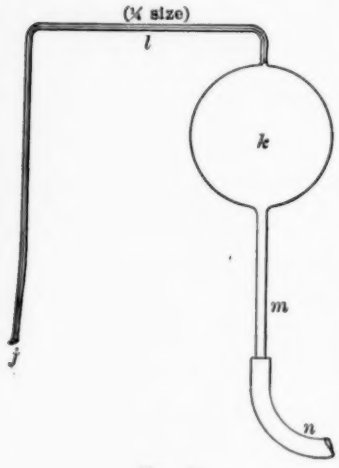


Fig. 11.

Mercury was poured into this reservoir, and it was raised until the mercury poured out of the orifice, *o*, in the capillary tube, which was plunged into a beaker of water which had been kept boiling for half an hour. The reservoir was then lowered, and the boiled water rushed into the bulb, *k*. On again raising the reservoir, this water was expelled, and boiled water was then introduced in its place. This was repeated seven or eight times, when the mercury reservoir was raised and the beaker removed. The water spurted out of *o* in a rapid stream, so that no air could pass back into the bulb. The point *o* was then dipped into the mercury of the funnel, and as the water was ejected from the capillary tube it floated up and occupied the upper portion of the inside of the umbrella. This was kept in position during filling by a set of three strout iron wires fixed in a wooden board, which was fixed on the rim of the funnel so that the lower ends of the wires pressed on the flat top of the umbrella. This was necessary because the umbrella was only stable when it contained more than a certain quantity of water. The point *o* was so made that although the water was projected upwards, the point offered no projection for the rim of the umbrella to catch upon, so that when sufficient water had been introduced, the tube could be removed.

The water used in these experiments was prepared from ordinary distilled water by redistilling in a new block-tin still, the earlier products of the second distillation being rejected.

TABLE IV.

I. Experiment.	Equilibrations. II. Water at 0°C.	III. Ice at $-t°$ C.	IV. Temp. $= -t°$ C.	V. W by subtracting 5.886 from Numbers in Column II.	VI. Buoyancy of Umbrella at $-t°$ C., from Fig. 10.
1	(*a*) 646.225	(*b*) 709.350	− 1.95	640.354	5.930
	(*e*) 646.255	(*c*) 709.675	−10.02		6.112
	Mean of (*a*) and (*e*) = 643.240	(*d*) 709.300	− 1.01		5.909
2	(*a*) 860.055	(*b*) 945.235	− 2.90	854.169	5.952
		(*c*) 945.100	− .37		5.894
3	(*a*) 822.495	(*b*) 903.450	− 8.37	816.609	6.074
		(*c*) 903.080	− 1.02		5.909
4	(*a*) 792.690	(*b*) 871.060	− 2.60	786.804	5.945
		(*c*) 870.769	− 6.59		6.034
		(*d*) 870.580	− 3.15		5.957

I. Experiment.	VII. I by Subtracting Numbers in Column VI from Those in Column III	VIII. $K = \frac{I}{W}$.	IX. $K+q$. $q = .07938$.	X. $h-t$.	XI. $i_{-t} = \frac{qh_{-t}}{K+q}$
1	703.420	1.09849	1.17787	13.6004	.916603
	703.563	1.09871	1.17809	13.6204	.917780
	703.391	1.09844	1.17782	13.5981	.916487
2	939.283	1.09965	1.17903	13.6028	.915863
	939.206	1.09956	1.17894	13.5965	.915509
3	897.376	1.09890	1.17828	13.6163	.917355
	897.171	1.09865	1.17803	13.5981	.916324
4	865.115	1.09953	1.17891	13.6020	.915903
	864.726	1.09904	1.17842	13.6119	.916950
	864.623	1.09890	1.17828	13.6034	.916487

After filling, the funnel and its contents were allowed to cool; the funnel was then closed with the closing plate as already described, and was placed in the apparatus, Fig. 9, the boxes having been previously washed out to get rid of the salt from the freezing

mixture. The mercury to seal the top was poured into the collar, and the funnel was then leveled. The jar and boxes were filled with ice, and the equilibration of the water was performed after waiting about eighteen hours.

The ice in the boxes was then exchanged for freezing mixture, the glass jar was emptied of its ice, and the metal vessel g was filled with freezing mixture and put into the jar. After leaving for two days, the ice was equilibrated, and this could be done at different temperatures, as the ice very gradually rose in temperature. The ice could then be examined, or a new equilibration of the water could be obtained, by allowing the ice to melt and bringing the whole again to 0° C.

The results of the different equilibrations and the computation of the density of ice for different temperatures below 0° C. are set out in Table IV.

In this table the letters prefixed to the equilibrations in columns II. and III., indicate the order in which these numbers were obtained. In finding the numbers in column IX., the value of q is only needed to four significant figures; but to compute the density of ice at different temperatures (column XI.) this same quantity q is .0793829.

The whole of the equilibrations performed with the final form of apparatus are given in the table. An improvement of the filler was introduced subsequently and anomalous results were obtained for the only experiment carried out. This was traced to the fact that the wire had become damaged during the filling, and the results for this experiment were rejected.

In the case of the first experiment the water was equilibrated at the beginning and end, and the mean value was used for computation. The ice in this experiment was not examined. In experiments 2 and 3 the ice was taken out after the last equilibration and was found to be free from milkiness and air bubbles, but it had fine circular cracks running round it concentric with the central stem of the umbrella.

The fourth experiment was conducted differently from the others, and the values obtained proved unmistakably that the same specimen of water may assume different densities on freezing. After the

value b had been obtained, the ice was permitted to melt either partially or completely. A fresh freezing mixture was put in the apparatus, and two subsequent equilibrations of the new specimen of ice were performed. The second specimen of ice had a considerably greater density than the first although it was made from identically the same water.

The results given in column IX. are plotted in Fig. 12. The points are marked with numbers indicating the experiment. The unbroken

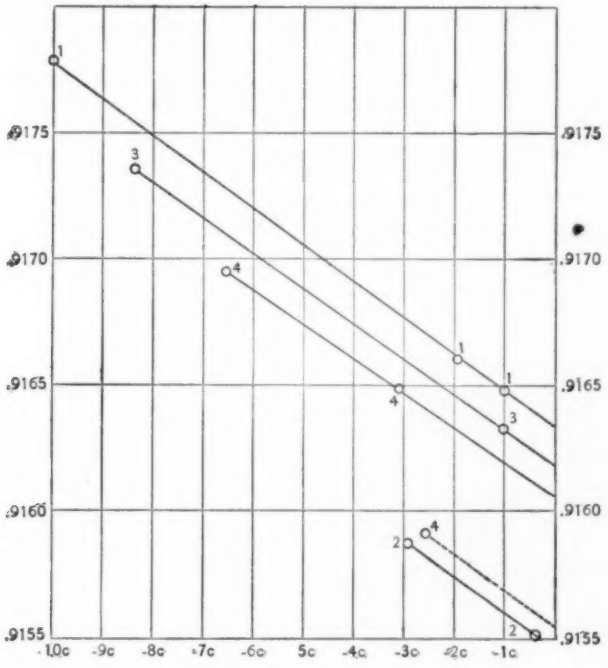


Fig. 12.

straight lines drawn through the points give us, by extrapolation, four values for the density of ice at 0° C., while a broken line drawn parallel to the straight line for Experiment 2 through the point given by the first specimen of ice in Experiment 4 furnishes a fifth value. The numbers thus obtained and the weights to be assigned to them in computing the mean are set out in Table V., the weight assigned in each case being equal to the number of equilibrations of the ice.

TABLE V.

Experiment.	Density of Ice at 0° C.	Weight Assigned.
1	.916335	3
2	.915460	2
3	.916180	2
4	.915540	1
	.916060	2
Weighted Mean.	.9160	

We thus obtain .9160 gramme per cu. cm. as the density of the ice at 0° C.

This result depends upon the assumption that the density of ice is a linear function of the temperature on a mercury-in-glass thermometer. Systematic error in the thermometer, so long as the zero is correct, is eliminated. The result likewise depends on the values assumed for the density of water and of mercury at zero, but is independent of the absolute mass of the weights employed.

THE COEFFICIENT OF CUBICAL EXPANSION OF ICE.

The errors of the thermometers are, however, involved in computing the coefficient of cubical expansion which also depends upon the particular law of dilatation of mercury used, but these errors are probably such as will not affect the result to the accuracy with which it is given below. The four values which can be found from the data available are set out in Table VI.

TABLE VI.

Experiment.	Coefficient of Cubical Expansion.	
1	.000155	Mean .000152
2	.000152	
3	.000153	
4	.000148	

COMPARISON OF RESULTS.

The value .9160 for the density of ice at 0° C., is lower by two parts in 10,000 than the mean of the results obtained by Plücker and Geissler, Bunsen and Nichols. It is 1 part in 10,000 less than

the mean of Nichols's values, but it is 7 parts in 10,000 lower than Bunsen's value. The value .000152 for the coefficient of cubical expansion is 4 per cent. lower than that of Plücker and Geissler, the last published value for the directly determined cubical coefficient of artificial ice. It is 5 per cent. lower than the mean value given in Table II.

Conclusion.

The results of this determination of the density and coefficient of cubical expansion of ice are, that Nichols's value for the density is confirmed, and that Bunsen's value is probably too high; but as the same specimen of water can freeze into specimens of ice having different density, the use of the Bunsen ice calorimeter in absolute determinations must be limited to an accuracy of probably about 1 in 1,000.

The coefficient of cubical expansion seems to be 4 or 5 per cent. less than the mean of previous determinations.

The expenses of this research have been in part defrayed by a Government grant from the Royal Society, and in part by the Cavendish Laboratory. I wish to thank Professor J. J. Thomson, F.R.S., for his kind encouragement, and my thanks are also due to Mr. Griffiths, F.R.S., through whom I was led to undertake the investigation.

AN EXPERIMENT RELATING TO THE APPLICATION OF LAGRANGE'S EQUATIONS OF MOTION TO ELECTRIC CURRENTS.

BY WILLIAM S. DAY.

THE experiment described in this paper was analogous to one made by Maxwell, an account of which is given in his Treatise on Electricity and Magnetism, Vol. II., Section 574. Maxwell's experiment was made for the purpose of determining whether or not any part of the kinetic energy of an electric current could be expressed by a term depending on the product of an ordinary velocity and an electric current. The velocity he considered was a velocity of the wire carrying the current in the direction of its length, and Lagrange's equations of motion show that if the term referred to exists, whenever a current is started or stopped in a wire, there would be an impulse acting on the wire in this direction. As Maxwell states, no evidence of any such effect has ever been discovered. The experiment described here was for a similar purpose, the difference between the two experiments being that in this experiment the mechanical coördinate considered is one such that its velocity means a rotation of the wire around its axis, the corresponding impulse when the current is suddenly started or stopped being an impulsive torque, if the term in question exists. No evidence of the existence of such an effect was discovered, but a maximum limit to its value was obtained, which is given on page 161.

In the application of Lagrange's equations of motion to electric currents which Maxwell made, and by means of which he so beautifully deduced the laws of electromagnetic induction, he considered that the state of a system of electric currents could be given by two kinds of coördinates, the ordinary mechanical and the electrical coördinates. The electrical coördinate he chose was one such that its velocity, or rate of variation with the time, expresses an electric current. A single linear circuit having only one degree of ordinary

mechanical freedom, can therefore be considered as a system having two degrees of freedom, one mechanical and one electrical. In the most general case the kinetic energy of a system having two degrees of freedom is a homogeneous quadratic function of the velocities of the two coördinates which correspond to the two degrees of freedom, and can be expressed as follows:

$$T = \tfrac{1}{2} I\dot{x}^2 + K\dot{x}\dot{y} + \tfrac{1}{2} L\dot{y}^2 \qquad (1)$$

in which T is the kinetic energy, x the mechanical coördinate, y the electrical coördinate, $\dot{x}$ and $\dot{y}$ being the time-derivatives of these. $\dot{x}$ is an ordinary velocity and $\dot{y}$ is a current. I, K, and L are coefficients which are frequently but not necessarily constants. I corresponds to a mass, L is the self-induction, and K is the coefficient whose existence it was the purpose of Maxwell's experiment to determine. In his experiment there was a circular coil of wire suspended so that it was free to rotate around its axis, that is so that each element of the wire could move in the direction of its own length. The velocity of his coördinate therefore expressed such a motion and Lagrange's equations show that if K is not 0, whenever a current is suddenly started or stopped in the wire it would experience an impulse tending to change the coördinate x. When an electric current in a wire is considered as something analogous to a flow of water through a pipe, such a coördinate is the natural one to choose. The advantage of considering the energy of a magnetic field as kinetic energy of rotation, the lines of magnetic force being the axes of the rotation, or vortex lines, while a current is a motion of translation—instead of considering the current as being the rotation while the energy of the magnetic field is kinetic energy of translation, the magnetic force being a velocity, also leads the mind to the same idea. But in spite of all that has been done by Maxwell, J. J. Thomson, Drude, Kelvin, and others in the way of constructing hypotheses, we are still extremely ignorant of the true mechanical explanation of electric currents and phenomena, and it seemed to the author that in applying Lagrange's equations to the phenomena we should not be content with discussing only one kind of coördinate, but we should consider every one that was geometrically possible and was not contradicted by known facts. Imagine

a closed electric circuit which is symmetrical around a straight line, a straight wire for example in the axis of a cylindrical shell with circular discs connecting the wire and shell at the ends. In a circuit of this kind, owing to the symmetry of the arrangement, apparently the only two mechanical coördinates which could properly be considered in this connection are that which Maxwell considered and the one taken account of in the present experiment.

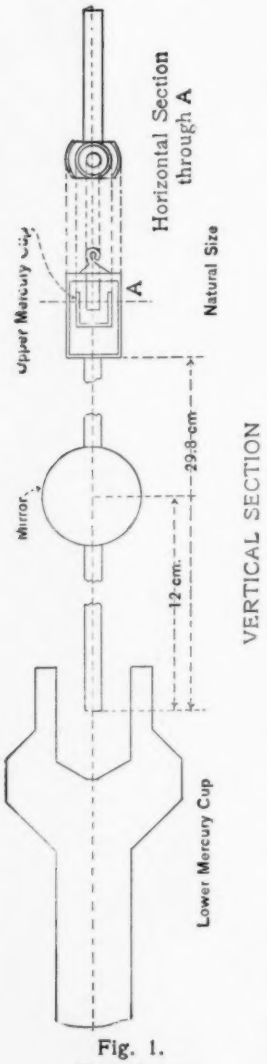


Fig. 1.
Natural size.

The experiment was tried in the following manner. A straight rod or wire of aluminum about 30 cm. long and 0.25 cm. in diameter was suspended by a quartz fiber about 12 cm. long in such a way that by means of mercury cups a current of 10 ampères could be sent through it at will in either direction. Figure 1 (which is of the actual size) represents how the mercury cups were arranged. The lower end of the aluminum rod was rounded and dipped into a mercury cup 1 cm. in diameter made of copper. The upper end of the wire was screwed to a nearly rectangular aluminum frame with a copper lug projecting downward in the inside of the frame from the top This dipped into a small mercury cup at the end of a copper bar extending horizontally from an upright standard. In order to avoid the capillary action which would tend to make the wire stick to the sides of the mercury cups, the latter were amalgamated while the wire was not. This caused a capillary repulsion between the cups and the suspended wire and kept the latter in the center of the cups. The quartz fiber could be adjusted at the top in azimuth and in height independently. The mercury cups could also be adjusted, and the instrument stood on three leveling

screws so that the system could be quickly and accurately brought to any desired adjustment. The motions of the wire around its axis were observed with the aid of a small silvered cover glass which was cemented to the wire and which reflected the image of an incandescent lamp filament on a scale.

The following disturbing forces which tended to mask the effect looked for had to be taken account of and reduced to as small an amount as possible.

The earth's magnetic field caused a force which acted on each element of the wire perpendicularly to its length and which, on account of the wire being not absolutely straight, in combination with the reaction of the mercury cups and the suspending fiber, produced a couple which tended to cause a constant deflection of the wire, the direction of which was reversed when the current was reversed. This was largely done away with by surrounding the wire with a piece of iron pipe 53 cm. long, 20.5 cm. inside diameter, and 0.7 cm. thick. The horizontal component of the earth's field was reduced in this way about 36 times, and any residual effect could be much further reduced by bringing up near the outside of the pipe the proper pole of a steel bar magnet.

A smaller source of disturbance was due to the magnetic field produced by that part of the current which was not in the wire itself. The importance of this was overlooked in designing the stand of the instrument and it might have been made considerably smaller. Its effect however with a little care was almost entirely done away with by bringing up near the wire a coil through which the same current passed. This effect tended to produce a steady deflection while the current was flowing, which was not reversed when the current was reversed.

A third disturbance was that due apparently to the magnetic effect of the induced currents in the iron pipe and other neighboring conductors. The upright stand of the instrument which supported the quartz fiber was made of a piece of glass tubing so that the chief effect must have come from the iron pipe. This might have been avoided by using a magnetic shield built up of laminated iron, but this was not done on account of the extra expense. The disturbance took the form of an impulsive deflection at starting and an op-

posite one at stopping the current, the direction being independent of the direction of the current.

The pressure of the mercury in the cups had the effect of reducing the free period of vibration of the wire from about 6 seconds to about 4 or even 2 seconds. The amount of reduction was different each time the wire was set up, but it remained quite constant as long as the adjustment remained the same. This change was evidently due to the effect of gravity acting on the not accurately straight wire in connection with the pressure of the mercury. It was regarded merely as if it were an increase in the elasticity of the quartz fiber, the moment of inertia of the wire being supposed to remain the same.

The effect of the well-known tendency of mercury to become covered with a film which holds light suspended bodies partly immersed in it as rigidly as if it were solid, was partly avoided by using only the purest mercury, having the parts that dipped into the mercury perfectly clean and free from amalgamation, and washing out the cups with mercury just before each experiment. The wire would usually vibrate quite freely with little damping when first set up if it were well immersed in the mercury, but this condition would not last very long, and the repeated turning on and off of a current as heavy as 10 ampères, after fifteen or twenty times, or much less sometimes, would increase the damping so that it was useless to proceed. It was then necessary to take everything down, clean up and start again. This peculiarity of mercury made the experiment very tedious.

The wire was protected from stray currents of air by means of a glass tube placed outside around the wire and the upright tube which supported the wire.

The mass of the suspended system was about 4.6 g., of which 4.2 was the mass of the straight part of the wire and 0.4 was the mass of the upper frame. The moment of inertia of the system was 0.069 g. cm.^{2} of which 0.024 was that due to the upper frame and 0.045 that of the straight part of the wire alone. These moments of inertia were determined experimentally by means of a small added tube whose moment of inertia was calculated from its mass and regular dimensions.

The calculation of the maximum value of the coefficient in equa-

tion (1) from the results of the experiment was carried out as follows. Let A be the value of this coefficient referred to 1 cm. length of the wire. Then Al would be its value for a wire of length l, that is, $K = Al$. Considering then the wire as having only one degree of mechanical freedom its kinetic energy would be

$$T = \tfrac{1}{2} I\dot{x}^2 + Al\dot{x}\dot{y} + \tfrac{1}{2} L\dot{y}^2 ; \tag{2}$$

I is the moment of inertia of the wire, L the self-induction, $\dot{x}$ the angular velocity, $\dot{y}$ the current. Forming Lagrange's equation for the force of type X,

$$X = \frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x}. \tag{3}$$

Since T is not a function of x, $\partial T/\partial x = 0$. Performing the differentiation and assuming that A is a constant as the simplest supposition,

$$X = I\ddot{x} + Al\ddot{y}. \tag{4}$$

The only external force, we will suppose for simplicity, is the torque of the fiber. This tends to decrease x and is proportional to it. Call it $-Nx$. Then the equation would be

$$-Nx = I\ddot{x} + Al\ddot{y}. \tag{5}$$

Multiplying by dt and integrating through the extremely short time, Δt during which the current is establishing itself, or dying out

$$-N\int_0^{\Delta t} x\,dt = I\int_0^{\Delta t} \ddot{x}\,dt + Al\int_0^{\Delta t} \ddot{y}\,dt. \tag{6}$$

As the average value of x during this time will be practically 0, the first member reduces to 0, and if, the system starting from rest, $\dot{x}_1$ is the angular velocity generated, and $\dot{y}_1$ the current which is started or stopped, the equation becomes

$$0 = I\dot{x}_1 + Al\dot{y}_1. \tag{7}$$

Therefore

$$A = -\frac{I\dot{x}_1}{l\dot{y}_1}. \tag{8}$$

This gives A in terms of known quantities and the angular velocity generated at the instant of turning on or off the current $\dot{y}_1$. This

angular velocity is measured by the deflection as in the case of the ballistic galvanometer. The system is vibrating under no force but the torsion of the quartz fiber and friction. Supposing its equation of motion now to be that of a system of only one degree of freedom, it would be

$$x = Be^{-\frac{Ft}{2I}} \sin \frac{2\pi t}{\tau}, \tag{9}$$

in which B is the maximum amplitude if there had been no friction, F is the coefficient of friction or retarding moment for unit angular velocity, and τ is the period of vibration. From this, when $t = 0$,

$$\dot{x}_1 = \frac{2\pi B}{\tau}. \tag{10}$$

The maximum elongation is, when $t = \tau/4$,

$$x_m = Be^{-\frac{F\tau}{8I}}. \tag{11}$$

Therefore

$$B = x_m e^{\frac{F\tau}{8I}} = x_m(1 + \tfrac{1}{2}\lambda) \tag{12}$$

in which $\lambda =$ the logarithmic decrement.

Therefore

$$\dot{x}_1 = \frac{2\pi x_m(1 + \frac{1}{2}\lambda)}{\tau} \tag{13}$$

and substituting this value in (8)

$$A = -\frac{2\pi I x_m 1(+)\tfrac{1}{2}\lambda.}{l\dot{y}_1\tau} \tag{14}$$

This is the equation to determine A from the observed quantities.

On one particular occasion, on April 14, 1902, the apparatus was adjusted more accurately than at any previous time. The period of vibration was about 2.2 seconds. On closing the circuit and allowing a current of 10 ampères to flow, there was a temporary deflection of 1.2 cm. on the scale, in the same direction, however, whatever the direction of the current. The magnetic forces acting on the wire for a steady current were so nearly balanced that it was estimated that if an impulsive torque due to a value of A had existed, it must have been capable only of producing a deflection on the scale smaller than 0.3 cm. or it would have been noticed. The

scale was distant 57.5 cm. It was, however, at an angle with the perpendicular to the radius drawn from the mirror whose cosine was $\frac{54}{57.5}$. The angle corresponding to the 0.3 cm. was therefore

$$x_m = \frac{0.3 \times 54}{2 \times (57.5)^2}.$$

λ was so small that for the purposes of this calculation it was neglected. Taking $I = 0.069$, $l = 30$, $\mathcal{Y}_1 = 1$ electromagnetic unit, and $\tau = 2.2$, we get

$$A = -\frac{2\pi \cdot 0.069 \cdot 0.3 \cdot 54}{2 \cdot (57.5)^2 \cdot 30 \cdot 2.2} = -0.000016.$$

The sign of course is of no consequence unless a deflection is actually observed. Call the result in round numbers 0.00002 This is the maximum limit to the value of A given by the experiment. The meaning of this is that if the coefficient A had this value in the sense in which it is used here, when a current of 10 ampères was decreased to zero at a uniform rate in exactly one second, there would be an apparent torque acting on each linear centimeter of the wire during this second equal to 0.00002 dyne. cm.

This effect could also be looked for from the point of view of the other coördinate. In the expression for the kinetic energy the two coördinates appear symmetrically, and therefore if an acceleration of the electrical type produces a mechanical force, an acceleration of the mechanical type should produce an electromotive force. That is to say, if a wire which was in very rapid rotation around its axis were suddenly brought to rest, there should be an electromotive force acting in the direction of the length of the wire, if A is not zero. If the wire is part of the circuit of a ballistic galvanometer, this momentary electromotive force could be observed, if it exists, just as in the case of momentary induced currents. The application of Lagrange's equations shows that the total quantity of electricity which would pass through the circuit would be

$$Q = -\frac{Al\dot{x}_0}{R},$$

in which l = the length of the wire, $\dot{x}_0$ = the original angular ve-

locity of the wire, and R = the resistance of the entire circuit. A rough calculation showed that this method did not give any certain promise of greater sensitiveness than the other method, using galvanometers of great sensitiveness and as great a change in angular velocity as could probably be produced. It did not seem worth while therefore to look for the effect in this manner also as long as there was so little theoretical reason to expect its existence.

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SPARKING DISTANCES BETWEEN PLATES FOR SMALL DISTANCES.

By R. F. Earhart.

IT is desired in this article to supplement a series of measurements made some two years ago, on the sparking distances between plates for small distances. (*Phil. Mag.*, Jan.,1901).

In the previous determinations, there was used an e.m.f. obtained from a bank of storage cells. In the present case measurements have been made in which the e.m.f.'s are periodic in character. Some measurements have been made on the effect of introducing capacity in circuit. The periodic e.m.f.'s here considered were taken from a transformer connected with the city lighting mains, and having a 60-cycle period. Dependence on station regulation is perhaps unfortunate but by proper choice of hours for experimentation little difficulty was encountered on account of fluctuation in potential. The conditions to be fulfilled in this problem are: (1) To measure a small distance accurately, (2) to measure a potential difference accurately.

In order to fulfill the first condition, recourse was had to an interferometer; measurements of distance being made in terms of wave-lengths of sodium light. The method of accomplishing this will be readily understood by consulting Fig 1. The electrical conditions require that one surface should be spherical. In this case a bicycle ball 2.52 cm. in diameter was mounted on a rigid support and insulated therefrom. The opposite surface (D') was mounted in a similar manner and attached to the movable carriage of an interferometer (C) which bears the mirror (M). The surface (D') could then be brought in contact with the spherical surface (D) and separated, the distance of separation being readily interpreted by the number of fringes passing the field during the course of separation. The method of determining contact between the sur-

faces was to include a galvanometer and a small e.m.f. (about .02 volt) and determine the point of contact electrically. This point can be easily determined to within ½ a fringe, *i. e.*, ¼ λ or approximately .15 micra. No effort was made to determine distances more accurately; for this lies well within the limit of accuracy of the e.m.f. determinations.

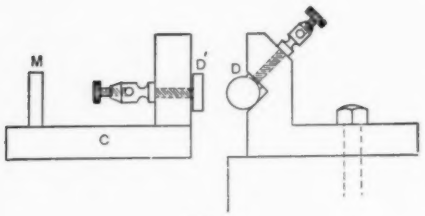


Fig. 1.

Correction has not been made for the sphericity of one surface for the distances measured are of the order of a wavelength of light, whence a radius of curvature of 1.26 cm. is relatively large.

The surfaces used were ground, polished and, in some cases, nickel-plated, approximating optical surfaces as nearly as possible. It is necessary that optical surfaces be approximated in order to obtain consistent readings.

The e.m.f.'s were obtained from a Westinghouse transformer 2000–100 volts, used as a step-up transformer. By suitable connections with the ordinary house mains an e.m.f. of 2,000 volts was available. This 2,000-volt current was passed through a bank of 20 lamps connected in series. These were mounted on a board, the terminals of each lamp dipping into a mercury cup as indicated in Fig. 2. It is thus possible to obtain a considerable range in potential difference by utilizing the drop in potential along the series. The P.D. between the terminals of each lamp, approximately of 100 volts, may be reduced by grouping lamps in parallels. An auxiliary board with suitable connections (not shown in figure) was used for this purpose; so that e.m.f.'s ranging from 10 to 2,000 volts in steps of 10 volts were available. These e.m.f.'s were measured by a Kelvin milleampère balance provided with suitable non-inductive resistances to reduce the flow through the balance below the safety limit.

The method of procedure, then, was to separate the plates a given distance and establish connection with the busbars (F, F)

Fig. 2. The P.D. between the busbars could then be varied by connections between the bars and the mercury terminals of the lamps.

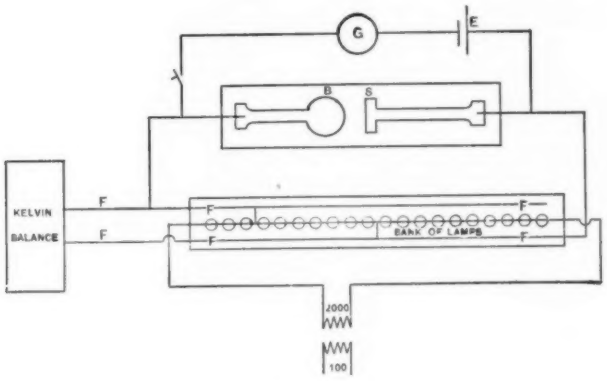


Fig. 2.

Since the Kelvin balance takes an appreciable current and is shunted across the lamps, this serves to reduce the potential between the lamp terminals, hence it is necessary to keep the balance circuit closed when applying an e.m.f. There being no inductive resistance in the circuit or a negligibly small one, the e.m.f.'s can be readily computed from the balance readings. In considering some of the results where capacity was included in the circuit some question as to the effect of the balance on the results arose. A set of Kelvin electrostatic voltmeters being available, these were connected across the busbars and measurements obtained almost identical with the balance both when the capacity was in the circuit and when the capacity was negligibly small. This instrument, like the balance, gives the square root of mean square values for a sine curve e.m.f.

Results.—A series of readings wherein nickel-plated surfaces were used are given in Table I. and represented graphically in Plate I. Readings given are those of maximum potential and obtained from square root of mean square values as given by the balance. This curve is almost identical with that obtained from static discharge, hence we conclude that a given potential is required to cause a spark to pass between such surfaces and that the character of the potential difference, whether static or perodic is unessential.

Nickel Surface.

Distance in Wave-lengths.	Potential in Volts.		Distance in Wave-lengths.	Potential in Volts.	
	1st Spark.	2d Spark.		1st Spark.	2d Spark.
1	72		15		515
1	120		20	392	520
1.5	128		25	403	510
2.0	137		30	403	598
2.5	242		35	414	573
2.5	250		35	441	555
3	256		40	547	656
3	270		45	555	671
3	248		50	590	743
4	337		55	585	750
5	350		75	700	820
5	300		85	750	865
6	321		100	785	865
6	307		100	786	1030
8	320		100	700	940
10	345		100	910	1070
10	347		150	974	1200
10	330		200	1100	1250
12	300	364			
15	371	500			

As with the static P.D., so in this case, there seems to be a change in the law at a distance of about three wave-lengths. At distances less than this the P. D. required to cause a spark to pass seems proportional directly to the distance of separation but above this point the relation between potential and distance follows a different law.

I have distinguished between what is termed the first and second spark. When a sufficient potential is established between the disk and ball a spark will pass, if now the e.m.f. be withdrawn the same potential will not cause the spark to leap the air gap a second time. It seemed possible that the discharge had torn off some of the plating, thus increasing the distance. To determining whether or not this was the case I reversed the motion of the interferometer carriage after the first discharge and counted the number of fringes passing the field during tne retrograde motion until contact was made. In the case of the plated surfaces and for a large number of measurements I found the point of contact to be unchanged.

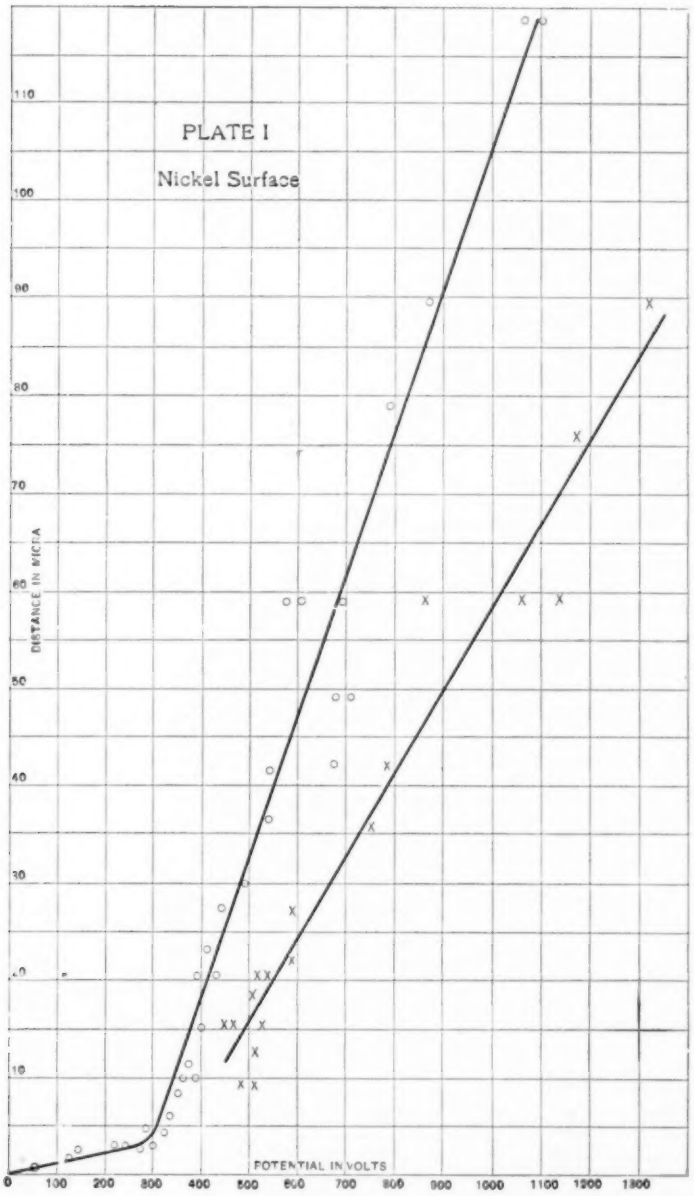


PLATE I
Nickel Surface
DISTANCE IN MICRA
POTENTIAL IN VOLTS
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
110
100
90
80
70
60
50
40
30
10

Fig. 3.

Steel Surface.

Distance in Wave-lengths.	Potential in Volts.		
	Capacity None.	Capacity 0.2 Microfarad.	Capacity 0.4 Microfarad.
5	90	125	135
8	225	200	155
10	255	230	165
12		290	234
15	298	286	303
20	305	320	330
25			392
30	345	360	410
40	410	420	530
50	496	490	584
60	520	545	659
75	580	600	728
100	670	825	872
125	790		
150	840		

A microscopic examination of the surfaces afforded a possible explanation. The nickel film seemed pierced by the discharge, the spark apparently seeking the ball below the nickel film. This discharge point had the appearance of a crater more or less regular in form, moreover the crater and adjacent nickel were coated with a film of iron oxide, probably carried out by the discharge. There were small projections at the edge of the crater but all fused and rounded down. Quite possibly, although the surface appeared initially smooth, there were minute roughened projections which the fusion by the discharge would round down. This, together with the slight coating of oxide, would explain the fact that a higher potential was required for second discharge. A number of early readings appeared to indicate some regularity for the second P.D. as defined above but a large number of readings proved this a fallacy. The readings given for first spark are shown in Plate I. by circles (o), for second spark by crosses (x). The potential required for second spark seems dependent on the character of the crater formed by the initial spark.

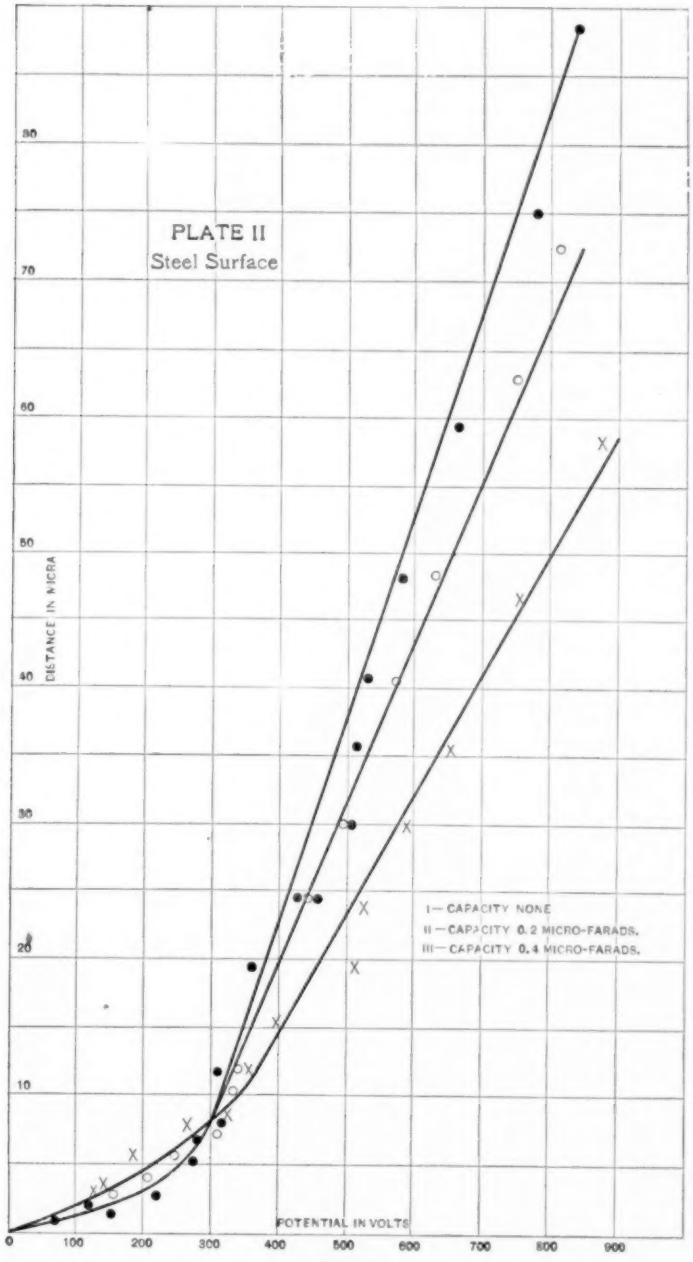


PLATE II
Steel Surface
DISTANCE IN MICRA
POTENTIAL IN VOLTS
I—CAPACITY NONE
II—CAPACITY 0.2 MICRO-FARADS.
III—CAPACITY 0.4 MICRO-FARADS.
0
100
200
300
400
500
600
700
800
900
10
20
30
40
50
60
70
80

Fig. 4.

In using steel surfaces, the general character of the curve agrees with the curve for nickel surface. A bicycle ball and disk ground and polished but not burnished were used. I was unable to obtain as satisfactory a surface in this case which probably accounts for the slightly lower potential required for a given distance.

In the case of no capacity curve, it was found that up to about 100 wave-lengths the potential for second spark was the same as for the first. Up to 15 wave-lengths the surfaces fused together on first discharge. Several measurements were made for a distance of 50 wave-lengths. It was found on pushing back the disk that the distance to contact had decreased from 10 to 12 wave-lengths, so that although the same potential as before was required the air gap was less. After a distance of 100 wave-lengths was reached the potential required for second discharge was greater than for the first. The oxidation and rounding down of the surfaces will, I think, account satisfactorily for all of these things.

The curves given in Plate II. are for steel surfaces. Curve I. representing values obtained with no capacity in the circuit, Curve II. when a capacity of 0.2 microfarad is introduced and Curve III. for 0.4 microfarad. Some measurements were made with a capacity of 1.36 microfarads which give values similar to those of Curve III. This particular condenser broke down at the higher potentials and for the present I have no means of checking the values, hence they are not tabulated. They seem to indicate that a farther increase in the capacity will not modify the result materially.

The effect of capacity seems to round out the elbow of the curve as it were. Below this point introducing a capacity seems to diminish the potential required for discharge, while above a limiting point a higher potential than before is required. The individual readings from which the curves are plotted are given in Plate II. Curve I. is plotted from values represented by a dot (.), Curve II. by a circle (o) and Curve III. by a cross (×). These values form only a small part of the data obtained but are not selected values. They represent one continuous run of readings, the other values showing fair agreement with these.

During the early stages of the experiment rather elaborate precautions were taken to exclude dust and water vapor, but in repeat-

ing a series of measurements these were dispensed with. It was found that the presence of dust and water vapor affected the results to a very slight extent, if at all. Most of the readings given are those in which no precautions were taken to guard against either dust or vapor and are under the conditions which ordinarily prevail in a laboratory with regard to temperature, etc.

I wish to express my obligations to the authorities of Ryerson Laboratory of the University of Chicago for the use of an interferometer with which these measurements were made.

ROSE POLYTECHNIC INSTITUTE,
TERRE HAUTE, IND., June, 1902.

THE GENERALIZATION OF GAUSS'S THEOREM.

BY S. J. BARNETT.

WITH one partial exception, mentioned below, theorists have hitherto demonstrated Gauss's theorem, so far as my knowledge goes, only for the case of a single homogeneous isotropic medium filling all space, and have then (implicitly) assumed it to hold true in all cases. The object of this paper is to point out the omission just mentioned and to establish in a logical manner the generality of the theorem.

First, consider the theorem as applied to electrostatics. In this case it may be stated as follows: *The electric flux outward across any closed surface surrounding a charge q* (rational units) *is equal to q;* or, for a single tube of induction, *the electric flux across every diaphragm of a given tube of induction is the same and equal to the charge at its ends.*

1. When the medium in which the charges are situated consists of a single infinite homogeneous isotropic dielectric, the law of inverse squares holds good in its simplest form, and the theorem follows logically, as in the common demonstration.

2. Now, by altering the charges and their configuration in the infinite dielectric just considered, the form of any tube of induction may be altered in any manner without altering the flux along it, called for brevity its *strength.* That is, the strength of a tube of induction is independent of its form.

3. If now dielectrics of different permittivity from that of the original dielectric, or conductors, or both are introduced into the field, the form of the portions of the tubes traversing what is left of the original dielectric will be altered; but, as just shown, this cannot alter their strength. Hence Gauss's theorem is valid throughout *any* region which contains a single homogeneous isotropic dielectric.

The validity of the theorem for the very important particular case in which the field contains any number of conductors and a single homogeneous isotropic dielectric can also be demonstrated as follows : Consider any number of equipotential surfaces, charged or uncharged, but *enclosing* no charges in an infinite homogeneous, isotropic dielectric. The surfaces may have any shapes, and any charges consistent with their being equipotentials. Throughout the field including these surfaces Gauss's theorem holds, by §1. When applied to the region enclosed within any of the equipotentials, the theorem shows that the electric intensity at every point in this region is zero. Hence the dielectric enclosed by any or all of the equipotentials can be replaced by any other dielectric or by a *conducting substance* without in any way affecting the external (and only) electric field or the validity of Gauss's theorem. This establishes the validity of the theorem for the case considered.

4. To establish the complete generality of the theorem for a region containing any number of homogeneous isotropic substances it is necessary to show in addition only that in passing from one dielectric into another a tube of induction does not alter in strength. To do this, consider the electric field between the plates A and B of a closed condenser containing two dielectrics, *1* and *2*, *1* being in contact with A only, and *2* in contact with B only. If the charge of A is q, that of B is $-q$, and there is no charge upon the interface between the dielectrics *1* and *2* (the charges due to contact being equal and opposite at any point). Applying Gauss's theorem to the region *1*, we find the total strength of all the tubes emanating from A to be q; and likewise, in the region *2*, we find the total strength of all the tubes terminating upon B to be q. Thus the total strength of all the tubes is unchanged in passing from A to B. And since this result is absolutely independent of the size or shape of the dielectrics or conductors, it follows that, however the field is divided up into tubes, the strength of *every* tube remains unaltered in crossing the interface.

5. By an obvious extension of what precedes, Gauss's theorem is seen to be valid also when the permittivity of the dielectric varies continuously from point to point instead of changing suddenly at distinct interfaces.

6. To extend the theorem to æolotropic media, we have only to remember that when the electric induction has the direction of any one of the three principal axes the phenomena are in all respects precisely the same as if the dielectric were isotropic. Hence for induction in these three directions Gauss's theorem holds good. But the induction at any point is always the vector sum of three such component inductions. Hence the theorem is valid for any direction of the induction.

In magnetostatics Gauss's theorem, equating the magnetic flux into the surrounding medium from a pole to the pole strength, is valid only for the ideal case of a concentrated pole, and then only when the equal flux *to* the pole through the magnet is neglected. For this ideal case the theorem may be generalized in almost the same manner as in electrostatics. In general the flux from a magnetic pole is very different from its strength.

The theorem is always assumed to be valid also in the general electromagnetic field, which may contain moving charges and poles and varying intensities. The justification of this assumption is of course the agreement of its important consequences with experiment.

In gravitation the ordinary proof of the theorem is general, since the gravitational "permittivity" or "inductivity" appears to be identical for all kinds of matter.

The very inadequate generalization of Gauss's theorem to which reference was made in the first paragraph above is contained in Pellat's *Électrostatique non Fondé sur les Lois de Coulomb, etc.*, §33.[1] The generalization of the theorem given in this paper is wholly independent of Pellat's work, and was largely developed long before I had seen his article.

[1] Ann. de Chim. et de Phys. (7), V., 1, 1895.

ON THE CAVENDISH EXPERIMENT AND THE LAW OF INVERSE SQUARES IN ELECTROSTATICS.

By S. J. Barnett.

IT is not the object of this note to cast any aspersions upon the law of inverse squares, or upon the Cavendish experiment itself, or upon any legitimate deduction therefrom. "The law of inverse squares," in the words of Mr. Heaviside, "is true and always will be;" the Cavendish-Maxwell experiment was certainly well performed; and the experiment certainly proves that in a static field no electric charge resides within the substance of a conductor or upon the inside surface of a hollow closed conductor. It would appear, however, that a serious fallacy is involved in attempting to investigate the law of force in electrostatics by any such experiment.

To make this clear, consider two concentric spheres A and B, with radii a and b (a greater than b) respectively, drawn in an *infinite, homogeneous, isotropic, non-conducting dielectric.* Let the surface A be charged to a uniform surface density. Then it can be shown by well-known methods (exactly analogous to those applied by Newton and Laplace to the region within a homogeneous spherical shell of gravitating matter), that, if the law of force is the law of inverse squares, there will be no electric field within the sphere A; but that, if the law of inverse squares is not obeyed, there will be a radial electric field within as well as without A, and therefore an electromotive force directed from A to B or from B to A. But B will have no charge, being perfectly insulated from A. (The divergence or convergence of the electric field within A does not now indicate the presence of an electric charge, Gauss's theorem, or Poisson's equation, which equates the divergence of the electric displacement to the electric volume density, being true only when the law of inverse squares is followed.) To reduce the electromotive force from A to B to zero, B must be charged to a uniform electric sur-

face density, whose sign and magnitude can be calculated in terms of that of A when the law of force is given. Or, if we know that A and B are so charged that the electromotive force from A to B is zero, and if we know the ratio of the charge of B to that of A, we can calculate the law of force. See Cavendish, Electrical Researches, §§ 232–235; and Maxwell, Electricity and Magnetism, § 74.

Now Cavendish and Maxwell have implicitly assumed that the above mode of reasoning is valid when the spheres A and B are the surfaces of *conductors* and the sphere A is charged externally while in metallic contact with B, which receives no charge unless by conduction from A.

But the famous experiments of Faraday with hollow conductors, of one of which the experiment of Cavendish is only a particular case, have shown conclusively that the substance of a conductor cannot support a static field, and that a hollow closed conductor screens perfectly the region within it from all external electrostatic effects, *i. e.*, all electrical effects persisting after the external field has become steady. There is, moreover, not the very slightest reason to suppose that this property of a conductor has anything whatever to do with the law of force. Indeed, it seems perfectly certain that the law of force depends only on the nature of the electric displacement in a *dielectric* and the three-dimensional nature of space (if the displacement is continuous, or the electric flux along a tube of displacement constant, and the dielectric homogeneous and isotropic, the law of inverse squares is an immediate deduction). Thus, no matter what the law of force, there is no reason to expect to find an electric charge or an electric field within the substance of a conductor or on the inner surface of a hollow closed conductor in a static field. It therefore follows that the experiment of Cavendish is not conclusive (so far as the law of force is concerned).

The fact that in a region containing a homogeneous isotropic dielectric and conductors the electric intensity at any point can be computed on the assumption that the permittivity, or dielectric constant, of the conductors (and the volumes enclosed thereby, if the conductors are hollow) is equal to that of the dielectric does not in any way invalidate the above argument. For this fact is a direct

consequence of the validity of the law of inverse squares (with its corollary, Gauss's theorem) in the dielectric, there being, when this law is obeyed, no electric field within any equipotential surface enclosing no charges, and it being therefore wholly immaterial what substance or substances such a surface encloses. For the sake of computation, the substance within every such surface can be imagined replaced by a medium with the same dielectric constant as that of the external dielectric, when the medium filling all space becomes electrically homogeneous and the law of inverse squares can be applied. If the law of force differed from that of the inverse square, every equipotential surface enclosing no charges drawn in a region filled with a dielectric would contain an electric field, and the dielectric within such a surface could not be replaced by a conductor without reducing this field to zero and therefore disturbing the equilibrium of the external field. Thus in this case the intensity could not be computed by assuming the conductors, if present, replaced by an additional portion of the external dielectric.

What precedes illustrates the importance, not always recognized, of inserting the permittivity, or dielectric constant, in the expression for the law of force, the force for given charges being inversely proportional to the permittivity whatever the law of its variation with distance. When the medium filling up the whole field is not electrically homogeneous and isotrop c the facts cannot be expressed by a simple law.

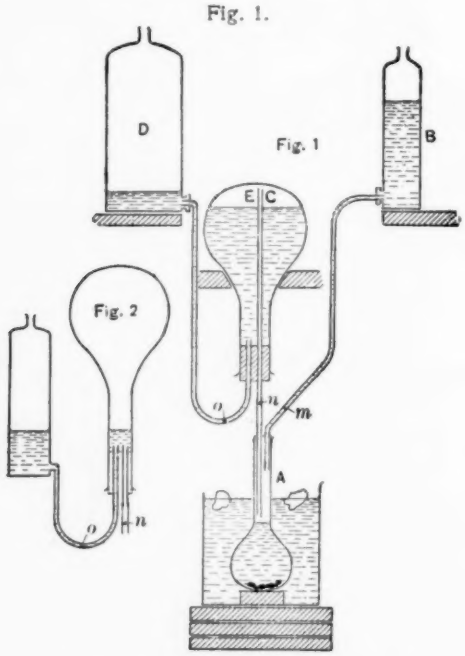
AN ACETYLENE GENERATOR.

BY H. A. RANDS.

THE acetylene generator here shown is a very simple and convenient arrangement for laboratory use where a small quantity of the gas is desired, free from air and as pure as the carbide used will permit.

In principle it is based on the work of Matthews who used alcohol to start the reaction slowly and with little evolution of heat.

Fig. 1.



As used it consists of a 100-c.c. flask *A* into which is fitted a two-hole rubber stopper, through each hole of which is passed a glass tube, one connecting by rubber tube to aspirator bottle *B*, and the other by short rubber connection to glass tube *E* leading up into

flask *C*. Flask *C* is also provided with a two-hole rubber stopper through one of which is the tube *E*, and through the other the short tube connected also by rubber tube with larger aspirator bottle *D*. In each connection is a valve or pinch-cock *m*, *n*, *o*, and, as a precaution, the stopper in flask *C* is retained by a wire catch similar to those used on soda water bottles.

Operation.—Flask *C* is filled with water, preferably recently boiled, from *D*, inverted and placed in a rest provided for it. Aspirator bottle *B* is partly filled with water, and both it and tube *E* are connected with stopper of flask *A*, stopper not being in the flask. To insure absolute expulsion of air valves *m*, *n* and *o* are fastened slightly open so as to leak a very little. Flask *A* is then filled to the very top of neck with 95 per cent. alcohol into which are dropped a few pieces of calcium carbide; after which the flask is raised, the stopper pressed tightly in place and flask supported at desired height on convenient blocks. Valve *m* is then closed tightly and *o* and *n* opened full. The reaction will be very slow at first and the gas first formed will collect at top of *A*; the pressure gradually forcing some of the liquid up into *C*. This will continue until foot of *E* is reached when bubbles will pass up into *C*, forcing some of the water out as shown. The reaction may be hastened at any time by opening *m* for an instant, allowing water to replace some of the alcohol. Should the reaction become very rapid so that the small amount of liquid in *A* becomes heated the blocks may be removed and a beaker containing water and a few bits of ice may be raised up under it as shown, which will insure coolness.

When a sufficient quantity of acetylene has been obtained *n* is closed, *A* removed and the water about stopper will form a perfect seal. If a measured quantity is desired *c* may be a graduated flask, *e. g.*, 250 c.c. and if when half full *D* be lowered so that surface of liquid in it be kept level with descending maniscus in *c* and *n* closed at proper time there will be obtained the desired quantity at atmospheric pressure. For this measurement it is best to have *E* short (see Fig. 2) as then, after *A* is removed, *o* and *n* may be opened and the water will force any remaining bubbles of gas out of *E* without in any way disturbing the gas above. This arrangement

[1] Matthews, Journal of American Chem. Soc., Feb., 1900.

was also found very convenient for clearing air from any apparatus connected to *c*, though, obviously to draw gas entirely from *c* it would be necessary to place in upright position.

The apparatus as described was used to generate two liters at one charging of *A*, but more could be obtained or larger flasks employed.

In charging *A* 5 grams were allowed for one liter of gas, though theoretically 64 grams calcium carbide (CaC_2) should give 22.4 liters acetylene (C_2H_2) or one gram give 350 c.c. The best carbide however will give but about 300.

CHEMICAL LABORATORY,
UNIVERSITY OF CALIFORNIA, April 10, 1902.

TEST OF THE LIQUID AIR PLANT AT CORNELL UNIVERSITY.

By Frank Allen and William Ambler.

THE Department of Physics recently installed in the basement of the main laboratory a plant for the production of liquid air for experimental purposes, and a number of tests have been made to determine its output and efficiency. The installation consists of a separate air compressor and liquefier, the former being belt-driven from an independent direct-current electric motor.

The compressor was built by the Norwalk Iron Works and is furnished with a Hampson liquefier made by Brin's Oxygen Co., Limited, of London. The compressor is of the four-stage type with air cylinders of 7¼-, 3⅞-, 2- and 1-inch diameters, and of 8-inch stroke. The cylinders are arranged in parallel on the same bed in the usual manner with inter-coolers between each pair.

The air is led into the compressor through the intake pipe which extends through the basement wall and to the roof of the building. The air first passes through a lime valve to rid it of dust and the greater part of the moisture. In the low pressure cylinder it is compressed to 43 pounds per square inch. From here it passes through an inter-cooler where it is reduced to room temperature. It then enters the second cylinder where it is compressed to 170 pounds and from which it passes into the second inter-cooler. From here it passes into the third cylinder and out into the third inter-cooler at 725 pounds pressure. Thence it is forced into the high pressure cylinder and is compressed to 2,600 pounds. After passing through the after-cooler to again reduce the air to ordinary temperature, it flows through a separator where the remaining moisture, oil and carbon dioxide are removed. Thence it passes to the liquefier where the pressure is kept at about 180 atmospheres. Connection to the liquefier is so made that it may be dis-

connected and the compressor used to furnish compressed air at any desired pressure within the capacity of the machine.

The motor, supplying power to the compressor, is a 30-H.-P., 500-volt, 4-pole, shunt-wound machine made by the General Electric Co., and has a nominal speed of 1,030 r. p. m. The mechanical power supplied to the compressor was found from the motor input and efficiency. This latter was found in the following manner. The belt was first taken off and the electrical energy necessary to drive it at no load and normal speed was determined. This value corrected for the copper losses at no load gives the total losses in the motor due to friction, hysteresis, eddy currents and windage. The copper losses at any load were computed from the currents flowing and the resistances of the armature and field, these latter having been determined by the ordinary fall of potential method. Given the electrical power input and the losses in the motor, the output, and hence the efficiency is readily found.

Readings of volts and ampères were taken by Weston instruments which had previously been calibrated by comparison with laboratory standards. The motor was run at about three fourths full load throughout the tests, at which output its efficiency was found to be 87.6 per cent. Therefore neglecting the small loss in the belt, the power supplied to the compressor was taken as 87.6 per cent. of the motor input.

The liquid air begins to flow from the nozzle of the liquefier within ten minutes from the time the compressor is started. Under the nozzle was placed a silvered Dewar bulb of two liters capacity supported upon one arm of a balance. The time taken to collect every two hundred grams of liquid was carefully noted until the bulb was full, when it was replaced by another. In the first test two liters were collected and removed, and a one-liter bulb of the same pattern substituted. The sudden rush of liquid air was, however, more than this vessel could stand, and it was shattered with explosive violence. This brought the first run to an abrupt end in a few minutes less than an hour.

The second test was made some time later with new purifying material in the separator. This, however, did not remove the atmospheric carbon dioxide as thoroughly as the original purifier, and

gave the liquid air a milky appearance due to the suspension of particles of the frozen gas. This seemed to make some difference with the action of the liquefier, and consequently the first run, though only half as long, is considered to be more representative of the performance of the plant. In addition to this, toward the end of the second run which lasted over two hours, the valves of the liquefier seemed to be a little clogged which perceptibly interfered with the rate of flow. During both runs, voltmeter, ammeter and speed observations were made at intervals of five minutes.

The quantity of air liquefied is usually about five per cent. of the amount passing through the system. The energy made available by this quantity of liquid air may be readily computed. Several determinations of the heat of vaporization have been made, which, however, differ widely. Dewar[1] found it to be 80 calories. D'Arsonval[2] gave the value as 65; while Behn[3] has found it to be 50.8. Behn's value has been confirmed by Dr. J. S. Shearer,[4] of the Physics Department, Cornell University. In this paper the heat of vaporization of liquid air is taken as 50 calories.

In addition to vaporizing the air, its temperature must be raised from the boiling-point (— 191°.4 C.) to the ordinary atmospheric temperature, that is through about 200°. The specific heat of air at constant pressure varies with the temperature, and also changes with the pressure. At 0° and atmospheric pressure it is 0.2375, and Linde[5] has computed its value to be 0.2467 at — 170° and the same pressure. In a test of this kind such a variation may be neglected in view of the approximate nature of some of the measurements.

The amount of heat necessary to raise the temperature of one gram of air through 200 degrees is about 47.5 calories. The total amount of heat, therefore, required to vaporize and raise to ordinary atmospheric temperature one gram of liquid air is 97.5 calories.

[1] Dewar, Scientific Uses of Liquid Air, Proc. Roy. Inst., Vol. 14, p. 393, 1894. Also in Chemical News, Vol. 71, p. 192, 1895.

[2] D'Arsonval, Compt. Rendus, 133, p. 980, 1901.

[3] Behn, Annalen der Physik, Vol. 306, p. 270, 1900 (1).

[4] Shearer, PHYSICAL REVIEW, Vol. XV., p. 188, 1902.

[5] Carl Linde, Process and Apparatus for Attaining Lowest Temperatures for Liquefying Gases, Engineer (Lond.), Vol. 82, p. 509, 1896.

Following are the results obtained:

TEST I.

Amount of liquid air produced,	2,700	grams.
Time,	55.5	minutes.
Rate of production,	2,919	grams per hour.
Energy made available by liquid air,	284,602	calories.
" " " " " "	879,420	ft. pounds.
Average power supplied,	18,881	watts.
Total foot pounds per hour supplied to motor,	50,129,055	
Efficiency of motor,	87.6	per cent.
Power supplied to compressor in foot pounds per hour,	43,913,052	
Efficiency of compressor and liquefier,	2.0	per cent.
" " plant including motor,	1.7	"

TEST II.

Amount of liquid air produced,	5,631	grams.
Time,	139	minutes.
Rate of production,	2,431	grams per hr.
Energy made available,	237,022	calories.
" " "	732,398	ft. lbs.
Average power supplied,	18,650	watts.
Total ft. lbs. supplied to motor per hr.,	49,515,750	
" " " " compressor,	43,375,797	
Efficiency of compressor and liquefier,	1.7	per cent.
" " plant, including motor,	1.5	"

In Test I. the energy supplied was 25.2 horse-power-hours, and as the production of the air was at the rate of 2,919 grams per hour the performance may be expressed as 116 grams per horse-power-hour. Taking the density[1] of liquid air as 0.932, the performance may be expressed as 124 c. cm. per horse-power-hour. In a descriptive leaflet issued by the Brin's Oxygen Co., with the Hampson liquefier, it is stated that with 3.5 indicated horse-power the liquefier had produced 1.2 liters per hour. The compressor used was a two-stage one. The performance is therefore about

[1] C. T. Knipp, On the Density and Surface Tension of Liquid Air, PHYS. REV., vol. 14, 1902, p. 75.

340 c. cm. This is about the same as stated by Travers in his book, "The Study of Gases." Corresponding figures for the Linde process are as follows[1]: With 3 horse-power 900 c. cm. of liquid air are produced per hour, or 300 c. cm. per horse-power-hour. In the paper referred to there is mentioned a plant under construction which is expected to produce 50 liters per hour with 120 horse-power. This will be a performance of about 420 c. cm. per horse-power-hour. D'Arsonval in the paper referred to mentions a Linde plant which has an output of 7 or 8 liters with an expenditure of 20 horse-power. It requires 35 minutes to liquefy the air. It has been also theoretically computed[2] that one horse-power-hour will produce 100 grams of liquid air, which is not far from the actual performance of the Cornell University plant.

It is evident, however, that the values which have just been discussed under Test I. and Test II. do not represent the true output of the plant. There is a constant evaporation not only from the surface of the liquid in the containing vessels, but also from the air streaming from the liquefier.

The loss due to evaporation should not properly be charged against the producing plant, but is due to the inefficiency of the receiving apparatus. Experiments were made to determine this loss as accurately as possible. The evaporation from the surface of the liquid was determined by weighing and found to vary greatly for different bulbs. The two-liter bulb lost 35 grams in 32.5 minutes which is at the rate of 65 grams per hour. The evaporation from the one-liter bulb was much less, being almost exactly 11 grams per hour for the period of four hours during which it was observed.

To determine the rate of evaporation from the stream of liquid air, we used a double-walled tin vessel with a tap at the bottom connecting with the inner part. This was filled with a known weight of air, and the outer part of the vessel was also filled with the liquid so as to reduce the evaporation from the inner to a negligibly small quantity. The liquid was then allowed to run out of the tap at

[1] Linde's Process for Producing Liquid Air, Engineering, vol. 65, 1898, p. 310. Also: Hardin, Liquefaction of Gases, p. 192.

[2] A. H. Neureuther, Liquid Air and the Power Required to Produce it; Street Ry. Review, May 15, 1899.

about the same rate as it comes from the liquefier into the one-liter bulb mentioned which had just been emptied of air, and was, therefore, at a low temperature. Making allowance for the evaporation from this, the loss by evaporation from the stream was found to be seven per cent. The gross output of the plant can therefore be determined by adding to the quantity collected the amount lost by evaporation.

The results obtained after making allowance for this loss are as follows:

TEST I.

Net rate of production,	2,919	grams per hr.
Gross " "	3,209	" "
Available energy,	312,877	calories.
" "	966,790	ft. lbs.
Gross efficiency of compressor and liquefier,	2.2	per cent.
" " " plant,	1.9	"

Performance is therefore 127 grams per h. p. hr.

TEST II.

Net rate of production,	2,431	grams per hr.
Gross " "	2,682	" "
Available energy,	261,495	calories.
" "	808,020	ft. lbs.
Gross efficiency of compressor and liquefier,	1.9	per cent.
" " " plant,	1.6	"

A run was also made with the valves of the compressor opened as far as possible to determine the power consumed by friction. This was found to amount to 11.5 horse power, or about one half of the total power required at full load. It was necessary to take into account the lower efficiency of the motor at this load, and it was also found impossible to open the valves completely, the gauge on the low pressure cylinder showing 15 pounds and the one on the second cylinder 12 pounds pressure per square inch. The normal speed of the compressor was 180 revolutions per minute.

Since the foregoing tests were concluded, a second liquefier of the same type has been added to the plant, and used simultaneously with the other. The output is thus about doubled, probably with the expenditure of more energy.

The plant has been in very frequent use, occasionally for three or four hours continuously, during the year, and the above results probably quite fairly represent the performance under practical conditions.

We desire to express our thanks to Professor E. L. Nichols for suggestions, and for kindly giving us the use of the plant for these tests.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY.

THE HEAT OF VAPORIZATION OF AIR.

BY J. S. SHEARER.

THREE observers have thus far placed on record values for the heat of vaporization of air. The first determination was made by Dewar[1] who found that it required about the same number of calories to vaporize a gram of liquid air as to melt a gram of ice. Quite a different value was found by Behn,[2] who gives 40 calories per gram, and later by D'Arsonval[3] who states that it requires 65 calories.

The relatively large variation in these results suggested the advisability of another determination. The writer was unable to find a statement of the method used by D'Arsonval but those of Dewar and Behn depend on the specific heat of mercury in the former case and of aluminium in the latter. This method hardly allows of a continuous and fairly uniform vaporization which could be readily controlled and be entirely independent of the thermal properties of any substance. It was suggested that the energy required could be easily supplied by an electric current and measured with considerable accuracy. For this purpose a small spiral of german-silver wire (C), Fig. 1, was wound and soldered to copper-lead wires. A pair of copper wires were also attached to the ends of the heating coil and connected to a large Weston voltmeter (V). The heating current also passed through a standard 10 ohm coil (R) made by Hartman and Braun from the terminals of which wires were led to the voltmeter. A double throw-switch (K_2) enabled alternate readings of the drop in potential in the heating coil and standard resistance to be taken. The two sets of lead wires were passed through a glass tube which was inserted in a rubber cork fitting closely in the neck of a

[1] Proc. Roy. Inst., 14, p. 393; Chem. News, 71, p. 192.

[2] Ann. der Phys., 1–1900, p. 270.

[3] Comptes Rendus, 1901.

Dewar bulb. The upper end of the glass tube was closed with rubber cement, and in order to be certain that it was air-tight a piece of soft-rubber tubing was wired to the glass and also to the wires and again sealed with hot cement. The air from the bulb passed through a second glass tube to a heavy piece of pressure tubing and was measured by an Elster gas meter (*M*). On the way to the meter it passed the bulb of a thermometer (*T*) which was sealed into a glass tube and a small tube was inserted so that samples of the gas could be taken for analysis.

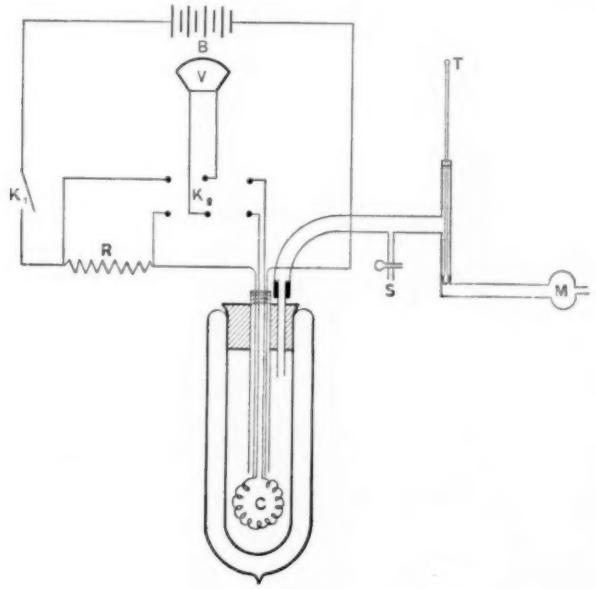


Fig. 1.

When no current was passing in the heating coil the rate of evaporization was observed, and this enables one to eliminate the heat supplied by conduction, etc., from that due to the current.

In order to get the density of the mixture given off samples were analyzed at frequent intervals by Mr. J. G. O'Neil to whom the thanks of the writer are due for much assistance in making the readings.

The first observations were made with air fresh from the liquefier and contained in a large Dewar bulb.

HEAT OF VAPORIZATION OF AIR.

Voltmeter No. 131, large Weston standard.

Voltmeter resistance 549 + 2196, *i. e.*, five times normal, 3-volt scale, 150 div., changed to 15-volt scale.

Current by drop around 10 ohm. Hartmann and Braun standard resistance.

Gas measured by Elster meter.

Temperature, Baudin thermometer.

Analysis, pyrogallol absorption. [Mr. O'Neil.]

RUN No. 1. APRIL 7, 1902.

Barometer 742.				Analysis 21.8 per cent. O.		
Hr.	Min.	Sec.	Meter.	Current	Volts.	*t*
3	15	52	160	0	0	18°
3	20	16	70			
3	43	2	135	112.8	76.4	17°
3	44	1	138			
3	46	11	144	112.5	76.2	
3	47	20	147			
3	48	30	150			
3	49	40	153			
3	50	50	156			
3	51	25	159	112.2	76.1	
4	6	2	15	112.1	76.0	17°
4	9	8	90	00	00	
4	13	52	150			%O 21.8

Meter [1 revolution = 3 liters, 180 div. on scale].
Small pointer registers liters.
Small pointer read while current was passing.

Total volume of gas while heating,	60 liters.	
Time of run,	1380 sec.	
Mean rate no current,	.0051 liter per sec.	
Total loss due to conductivity, etc.,	7 l.	
Vapor due to current,	53 l.	
Mean current,	.762 amp.	
Mean voltage,	11.24 volts.	
Log. .762,		1.881955
Log. 11.24,		1.050766
Log. 13 80,		3.139879
Log. .24,		1.380211
		3.452811
Log. 53,		1.724276
		1.728535

Calories per liter (742 mm. 17° C. 21.8 per cent. O) 53.52. One liter at 742 mm., 17° = .919 l. At 760, 0°, .919 l. at 0°, 760, 21.8 per cent. O = 1.188 grams.

Heat of vaporization per gram = 44.02.

Other results: 22.5 per cent. O, 45.4; 56 per cent. O, 50.57; 72 per cent O, 51.7.

When the greater part of the liquid had been evaporated the remainder was transferred to a cylindrical bulb and runs for higher percentages of oxygen were obtained. The readings when the liquid became very rich in oxygen were not entirely satisfactory, owing to the small amount of liquid remaining which perhaps did not completely cover the coil. There seemed however to be a distinct increase in the heat of vaporization as the liquid boiled away. The writer hopes to make a determination for pure liquid oxygen in the near future.

It will be seen from the data shown below that the results agree fairly well with those of Behn, and would seem to indicate that the heat of vaporization is not far from 50 calories per gram.

It may be of interest to compare the experimental results with those indicated by the well-known formula in thermodynamics, viz., $L = (V_2 - V_1)dp/dT\ T$. The curves of vapor tension for oxygen have been given by Olzewski, Estreicher and Baly. The rate of change of vapor pressure with temperature in the neighborhood of 90° A. from the results of Olzewski and Baly agree quite well, while a higher value is indicated by Baly. Assuming that Boyle's law holds with fair accuracy for this range of temperature, $V_2 - V_1$ may be computed. Using dp/dT from Olzewski's work, L is approximately 45, while Estreicher's observations give L as about 53.6. From Olzewski's results for nitrogen we find its heat of vaporization approximately 110. The careful determination of this constant for liquid nitrogen would be of considerable interest.

CORNELL UNIVERSITY, June, 1902.

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Two New Editions (*De Morgan, on the Study and Difficulties of Mathematics*, second reprint edition). Pp. vii + 288. Chicago, the Open Court Publishing Co., 1902. *Mach, The Science of Mechanics.* Translated from the German by THOMAS J. MCCORMACK. Second revised and enlarged edition. Pp. xx + 605. Chicago, the Open Court Publishing Co., 1902.

It is gratifying to note that there is a sufficient demand for works of the character of De Morgan's little treatise on mathematics and Mach's mechanics to warrant the issuing of new editions. All teachers of elementary mathematics should be encouraged to supplement their routine work by the careful perusal of certain books, such as Clifford's Seeing and Thinking, Pearson's Grammar of Science and this little essay of De Morgan's on the Study of Mathematics. Mach's well-known treatise on mechanics should, and for a similar reason, be in the hands of every teacher of physics. It contains, in addition to the usual subject matter of our best text-books on this subject, a wealth of historical and illustrative material commonly omitted for lack of space; and it presents in a form accessible even to readers of limited mathematical training many principles of mechanics such as D'Alembert's principle, Taylor's theorem, Gauss's principle of least constraint, and Maupertuis' principle of least action, with which too many of our teachers of elementary mechanics never really become acquainted. E. L. N.

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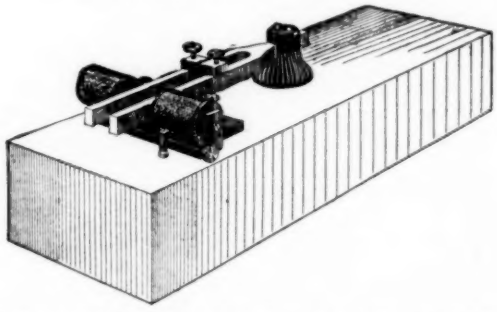

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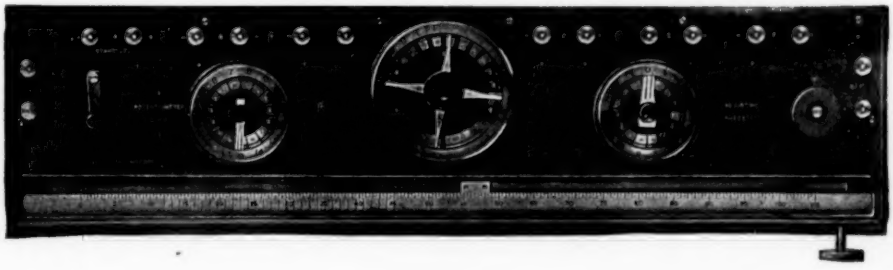

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